Submission Costs in Risk-Taking Contests

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Abstract

This paper investigates stochastic continuous time contests with a twist: the designer requires that contest participants incur some cost to *submit* their entries. When the designer wishes to maximize the (expected) performance of the top performer, a strictly positive submission cost is optimal. When the designer wishes to maximize total (expected) performance, either the highest submission cost or the lowest submission cost is optimal.

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JEL Classifications: C72; C73; D81; D82; D83

1 Introduction

There are significant incentives for managers in the mutual and hedge fund industries to outperform their fellows. Among other things, this competition is engendered by career concerns:[1] promotions are few and far between and managers

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[1]Chevalier and Ellison (1999) and Brown et al. (2001) are two empirical papers that document and explore career concerns in the mutual and hedge fund industries.

must outperform their fellows in order to be selected for such prizes. One natural way of modelling such competition is introduced by Seel and Strack (2013) (henceforth SS) who look at a continuous time, winner-take-all contest in which n agents compete by deciding when to stop independent Brownian motions with drift that are absorbed at zero. This captures the essentials of the scenario: returns are stochastic and private, and funds (or managers) can go bankrupt.

In this paper, we add a single, simple twist to the winner-take-all setting of SS.[2] Namely, we require that the contestants must incur some cost (a submission cost) to become eligible for the contest's prize. This is a natural feature of this paper's motivating example: an investment firm typically requires any promotion aspirant or prospective employee to fill out an application form and insists that any candidate disclose evidence of her successes.

Beyond competition between fund managers, there are other natural interpretations of the contests under study and submission costs therein. For example, academics choose which subfields to inhabit and what projects to pursue, where the prize is a successful grant proposal, a publication in a top journal, or a prestigious appointment. Grants and awards in academia mandate that researchers put together proposals, the preparation of which takes a considerable amount of time and effort. Similarly, graduate students in many fields prepare job market papers in order to secure one of the limited spots in academia and spend scores of hours in the job market season preparing their applications.

In these examples, the profession's esteem for the project should be seen as the value of the stochastic process. The running of the process is then the dissemination and exposition of the idea. Both positive and negative drift seem reasonable here: people get tired of seeing the same over-exposed paper, or a shaky project

[2] In fact, virtually all of our results can be established in the more general scenario in which agents decide when to stop non-negative time-homogeneous diffusion processes that are absorbed at 0, but the specific form of the Brownian motion with drift provides structure that eases the discussion and exposition.

may have a significant flaw that is more likely to emerge the more time it spends out. Conversely, the polishing of a paper that emerges as it is publicized may improve the contribution, at least on average.

At first glance, submission costs seem totally wasteful, especially given our assumption that they correspond to a pure destruction of surplus. However, submission costs affect the equilibrium play in the contest and any positive changes in this behavior could potentially outweigh the direct losses due to the costs. We take the perspective of a principal and look at two possible objectives: maximization of total (expected) output and maximization of the (expected) performance of the top agent. Naturally, the principal could either be a literal principal, e.g., the head of a fund overseeing the managers; or society, which benefits from better allocation of capital, innovations, and research.

Perhaps surprisingly, even though submission costs do not factor into the principal's utility (they are a negligible portion of her revenue), they may nonetheless benefit the principal. More specifically, when the drift is positive the total (expected) output is strictly increasing in the size of the submission cost. When the drift is negative this relationship is flipped and lower submission costs are better. Regardless of the sign of the drift, the (expected) maximal performance is increasing in the submission cost provided the submission cost is sufficiently small, and the maximal submission cost remains optimal for a positive drift. Contrary to received wisdom, the investment firm from the leading example may not wish to reduce "red tape" and lower (or eliminate) barriers to promotion or employment.

Submission costs produce two conflicting forces. On one hand, fixing agents' output distributions, submission costs discourage disclosure–why pay to lose? This is unambiguously bad for the principal. On the other hand, these costs also alter the equilibrium output distributions, and do so in a way that benefits the principal. At equilibrium each agent's strategy yields a continuous distribution over stopped values on $[0, \bar{x}]$ and places an atom on 0. An agent does not enter the contest if and only if she obtains value 0. This is not so bad for the principal; however, as this is the only value worthless to her. That is, *the negative force described above is completely negated* at equilibrium. When the drift is positive, the mass point placed on 0 also means that an agent's average submitted value increases (think of Bayes-plausibility), which is good for the principal by itself; and also generates longer tails, which is also beneficial.

1.1 Related Work

Several papers extend the results in SS in a variety of ways. Of special note is Nutz and Zhang (2022) (henceforth NZ) who study the effects of changing the prize structure in the model of SS, and find that more inegalitarian prize schedules lead to longer tailed distributions in managers' performance or output (stopped values of their respective processes)–greater risk taking–leading to higher average output when the drifts of the agents' processes are positive and higher maximal performance regardless of the sign of the drift. Also related is Seel (2015), who allows for heterogeneous loss constraints; Nutz and Zhang (2021), who look at a mean-field version of the game; and a trio of works, Feng and Hobson (2015), Feng and Hobson (2016a), and Feng and Hobson (2016b), who allow for more general diffusion processes, regret, and a random initial law, respectively.

Fang and Noe (2016) establish an equivalence between the stochastic contest of SS and a static game in which the contests choose randomizations over output that must satisfy a constraint on their mean (which game is itself a generalization of Wagman and Conitzer (2012)). This finding leads directly to our equilibrium characterization and uniqueness result. They also look at the effects of different (more or less equal) prize schedules, contestant heterogeneity and incomplete information, and other contest modifications (scoring caps, penalty triggers and localized contests) on equilibrium behavior and output.

There is also connection between these works and those papers that look at competitive Bayesian persuasion–see, e.g. Albrecht (2017), Boleslavsky and Cotton (2015), Au and Kawai (2020), and Hwang et al. (2018). For a binary prior and uncorrelated states, the competitive persuasion problem, the contest of Fang and Noe (2016) (with a performance cap), and the pricing game of Spiegler (2006) are equivalent. At the end of Section 3, we discuss this paper's findings in the context of competitive persuasion.

Naturally, this paper is also related to the substantial collection of papers that look at risk-taking contests more broadly. This group of papers includes Dasgupta and Stiglitz (1980), Bhattacharya and Mookherjee (1986) and Klette and De Meza (1986), who look at variants of an R&D contest; Hvide (2002), Hvide and Kristiansen (2003), Goel and Thakor (2008), Gilpatric (2009), and Fang and Noe (Forthcoming), who look at promotion contests; Basak and Makarov (2015), Strack (2016), Whitmeyer (2019), and Lacker and Zariphopoulou (2019), who look at contests between investment managers in financial settings; and Robson (1992), Hopkins (2018), and Zhang (2020), who look at contests for status and/or relative rank.

1.1.1 Entry Costs

There is also a literature that explores entry fees in all-pay contests. The first work in this area is Higgins et al. (1985), who study entry fees in a classical rent-seeking contest. In their seminal work, Moldovanu and Sela (2001) briefly study entry fees in all-pay auctions with incomplete information. There, the purpose of such fees are exclusionary: costs dissuade low ability types from entering. Importantly; however, they note that reducing entry is always suboptimal when agents are homogeneous.

Liu and Lu (2019) explore entry fees in Moldovanu and Sela's setting further and highlight the tradeoff between encouraging entry and effort. Specifically, winner-take-all encourages effort within the contest, which benefits the principal;

but dissuades entry, which harms the principal. In Section 4.1, we study entry costs in this paper's setting. Like in the standard all-pay environment, such costs have a pernicious effect on entry. Interestingly, they also have a negative effect on agents' experimentation and so are completely without merit from the principal's perspective. Fu et al. (2015), Ginzburg (2021), and Kaplan and Sela (2010) are three other works that study entry fees. The last highlights the benefit of entry fees on selection: entry fees can benefit a principal by encouraging the probability that high types win the contest.

All in all, entry fees in these all-pay contests trade off participation with effort and or selection. Our current surroundings have an endogenous-entry feature; namely, after experimenting, each agent has private information (her realization) and must decide whether to submit. As in these other papers, as we noted above, there is a direct negative effect of costs on participation/submission. However, as we also noted above, at equilibrium this negative effect is endogenously inconsequential. Moreover, the additional positive effect of submission costs on the agents' output distributions here is absent from the aforementioned works.

2 The Main Analysis

There are n agents $i = 1, 2, \ldots, n$ who participate in the following contest. Time is continuous, and at each point in time $t \geq 0$ each agent i privately observes the realization of a stochastic process

$$X_t^i = x_0 + \mu t + \sigma B_t^i \, ,$$

where $x_0 > 0$ is the initial value for each agent's process and each $\left(B_t^i\right)_{t \geq 0}$ is an independent Brownian motion. At any point in time, an agent may (privately) decide whether to stop the process and either submit the realization (and incur a cost $c \geq 0$) or decline to submit. If an agent's submitted output is strictly greater than the maximal submitted output of the other contestants, she obtains a prize normalized to 1. If there is a tie for first place, it is broken fairly (though this will not happen on the equilibrium path). If she does not submit her output, she gets a payoff of 0.[3]

We assume that the principal commits *ex ante* to the submission cost, and is limited to costs within some interval $[0,\bar{c}]$ with $\bar{c} \in (0,1)$. Moreover, following the literature on this problem, we assume that 0 is an absorbing boundary. This captures the fact that the fund (or fund managers) of this paper's motivation can go bankrupt. We also make the following parametric assumption, which ensures that agents' equilibrium stopping times are finite:

$$1 + n\frac{(1-\bar{c})\left(\exp\left\{-\frac{2\mu x_0}{\sigma^2}\right\}-1\right)}{1-n\bar{c}+(n-1)\,\bar{c}^{\left(\frac{n}{n-1}\right)}} > 0 \ .$$

This is always satisfied when $\mu \leq 0$.

A strategy for an agent i in the stopping problem is a stopping time τ_i. Equivalently, because it is only the distribution over stopped values that matters, this problem of choosing τ_i can be reduced to one of choosing an optimal distribution F_i over stopped values that is feasible, i.e., that can be induced by a stopping time. This problem of finding a stopping time to embed a probability measure is the well-known Skorokhod embedding problem.[4] Moreover, the set of feasible distributions is readily available for us to take "off-the-shelf." Let $\mathcal{F}$ be the set of feasible distributions and $s(x)$ be the scale function of the stochastic process X. For the Brownian motion with drift of this paper, the scale function is

$$s_B(x) = \begin{cases} \frac{\sigma^2}{2\mu} - \frac{\sigma^2}{2\mu}\exp\left\{\frac{-2\mu x}{\sigma^2}\right\}, & \text{if} \quad \mu \neq 0 \\ x, & \text{if} \quad \mu = 0 \ . \end{cases} \tag{1}$$

Then, from Theorem 2.1 in Pedersen and Peskir (2001) (see also Lemma 1 in SS, the discussion on p.25 of Feng and Hobson (2015) and Lemma 2.1 in NZ),

[3] Our results do not change qualitatively if a contestant is still eligible for the prize when she does not submit (more on this at the end of Section 3).

[4] Obłój (2004) surveys the literature on this problem.

Remark 2.1. The set of feasible distributions $\mathcal{F}$ consists of all distributions F supported on $[0,\infty)$ that satisfy $\int_{\mathbb{R}_+} s(x)\,dF(x) = s(x_0)$.

Note that when X is a martingale, the set of feasible distributions becomes those distributions F on $[0,\infty)$ that have mean x_0. Moreover, because the scale function is strictly monotone, as noted by NZ–see, e.g., the discussion preceding their Theorem 3.2–and Feng and Hobson (2015), it is without loss of generality to solve for the equilibrium (and verify uniqueness) in the driftless case.

This problem (without a submission cost) is solved in Fang and Noe (2016) (Theorem 1), and it is easy to see that the fee only alters the strategic interaction slightly. Indeed, observe that agents' distributions over values that they submit must be atomless and such that the payoff for each agent in submitted values is an affine function of the value. Accordingly, for $c > 0$, each agent must both submit and not submit with positive probability–the former because the distribution over submitted values must be atomless, and the latter because an agent always benefits by deviating and submitting if nobody else does.

It is also clear that if an agent does not submit value x' at equilibrium, then she does not submit any values $x \leq x'$. Thus, because a concavification argument eliminates all other possibilities, the only value that an agent does not submit at equilibrium is 0–her equilibrium distribution must place a mass point on 0. Moreover, as an agent's payoff as a function of her submitted value must be continuous at equilibrium–see, e.g., SS's proof of their Proposition 3 or the proof of Lemma 4 in Fang and Noe (2016)–she must be exactly indifferent between submitting 0 or not, i.e.,

$$F^{n-1}(0) - c = 0 \, .$$

Thus, the unique symmetric equilibrium when X is a martingale is for each agent to choose distribution

$$\tilde{F}(x) = \left(c + \lambda \frac{x}{x_0}\right)^{\left(\frac{1}{n-1}\right)}, \quad \text{on} \quad \left[0, (1-c)\frac{x_0}{\lambda}\right],$$

where

$$\lambda = \lambda(c) := \frac{1 - nc + (n-1)c^{\left(\frac{n}{n-1}\right)}}{n}.$$

The agent does not submit if and only if she obtains realization 0.[5] Substituting in the scale function, we generate the equilibrium distribution when the diffusion has drift:[6]

$$F(x) = \left(c + \lambda \frac{s_B(x)}{s_B(x_0)}\right)^{\left(\frac{1}{n-1}\right)}, \quad \text{on} \quad \underbrace{\left[0, \frac{-\sigma^2}{2\mu}\log\left\{1 - \frac{2\mu}{\sigma^2}\frac{(1-c)s_B(x_0)}{\lambda}\right\}\right]}_{:=[0,\bar{x}(c)]}. \qquad (2)$$

Proposition 2.2. *There exists a unique symmetric equilibrium. Each agent chooses the distribution F given in Expression 2 and does not submit her realization if and only if it is* 0.

When there are only two agents, it is easy to show that the unique symmetric equilibrium of Proposition 2.2 is in fact the only possible equilibrium, symmetric or otherwise.

Remark 2.3. Let $n = 2$. In the unique equilibrium, each agent chooses the distribution F described in Expression 2.

3 Comparative Statics

In this part, we follow Section 3 of NZ closely. Naturally, the mechanism used by the principal to effect change is different, yet we show that raising submission costs affects equilibrium play in the same manner as making the prizes more unequal. Of course, the prize schedule in this model is already as inequitable as possible,

[5]This is due to the necessary continuity of an agent's equilibrium payoff: there cannot be a positive probability of a tie at any disclosed value. Otherwise, an agent could do strictly better by experimenting a little longer, thereby breaking the tie in her favor.

[6]A previous version of this paper also studies the case in which X is exponential Brownian motion. In that specification, analogs of these results all hold.

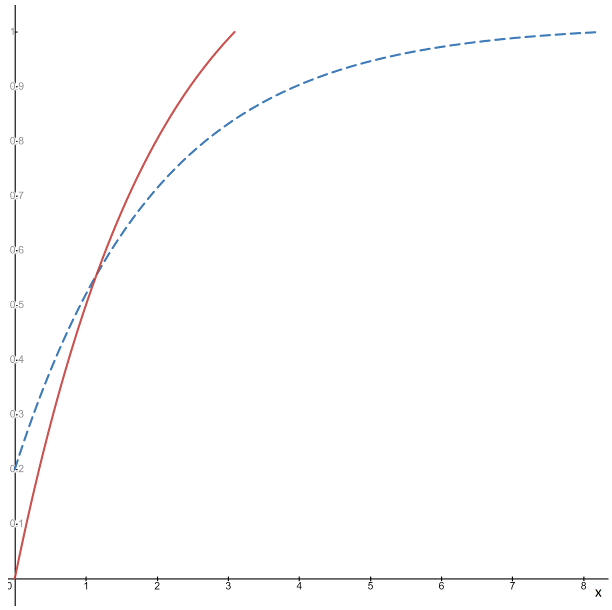

Figure 1: Equilibrium cdf for $c = 0$ (solid red) and $c > 0$ (dashed blue).

yet submission costs still can benefit the principal significantly. The proofs for this section may be found in Appendix A.

We say that two cdfs F and $\hat{F}$ are strictly single-crossing if there exists a point $x^* \in \operatorname{supp} F \cup \operatorname{supp} \hat{F}$ such that $F(x) > \hat{F}(x)$ for all $x \in [0, x^*)$ and $F(x) < \hat{F}(x)$ for all $x \in (x^*, \bar{x}(c))$ (with equality for all $x < 0$ and $x \geq \bar{x}(c)$ and at $x = x^*$).

Lemma 3.1. *If $c > \hat{c}$, the equilibrium distribution corresponding to $\hat{c}$, $\hat{F}$, is strictly single crossing with regard to the equilibrium distribution corresponding to c, F.*

This single crossing phenomenon can be seen in Figure 1. Next, we encounter the following theorem, which specifies the effect of submission costs on average equilibrium output.

Theorem 3.2. *Let $c > \hat{c}$ and F and $\hat{F}$ be the corresponding equilibrium distributions, respectively. Let $\phi : \mathbb{R}_+ \to \mathbb{R}$ be an increasing, absolutely continuous function.*

(i) If ϕ'/s' is increasing on $(0, \bar{x}(c))$, then $\mathbb{E}_{\hat{F}}[\phi(x)] \leq \mathbb{E}_F[\phi(x)]$.

(ii) If ϕ'/s' is decreasing on $(0, \bar{x}(c))$, then $\mathbb{E}_{\hat{F}}[\phi(x)] \geq \mathbb{E}_F[\phi(x)]$.

The inequalities are strict unless ϕ is an affine transformation of s.

For Brownian motion with drift, $s'(x) = \exp\left\{-2\mu x/\sigma^2\right\}$, and so $d/dx\left[\phi'(x)/s'(x)\right]$ has the same sign as $\phi''(x) + 2\mu/\sigma^2\phi'(x)$. Thus,

Corollary 3.3. $\mathbb{E}[\phi(x)]$ *is increasing in the submission cost when* $\mu \geq 0$ *and* ϕ *is convex and decreasing in the submission cost when* $\mu \leq 0$ *and* ϕ *is concave. These inequalities are strict if* ϕ *is not constant and* $\mu \neq 0$*.*

This immediately implies the following result concerning the effect of submission costs on the average output of each agent.

Corollary 3.4. $\mathbb{E}[X_\tau]$*, is strictly increasing in the submission cost when* $\mu > 0$ *and strictly decreasing in the submission cost when* $\mu < 0$*. Thus, the uniquely optimal submission costs, respectively, are* $\bar{c}$ *and* 0*.*

One interesting relative of this result concerns the magnitude of the improvement in output that can be brought about via a submission cost. Namely, for certain regions of the parameter space the total output with two contestants and the optimal submission cost exceeds the total output with three contestants and no submission cost.

Our final result of this section concerns the expected value of the contest winner's performance. As one might suspect, the expected value of the maximal output is increasing in the size of the submission cost when the drift is positive. Intuitively, when the principal cares about the maximum from a number of draws from a distribution, she prefers longer right tails. When the drift is positive, she also prefers such longer experimentation and so the effects act in synergy. On the other hand, when the drift is negative, the two forces are possibly countervailing since increasing the second moment sacrifices the first. However, it turns out that the latter is more important: the principal always prefers a small submission cost to no submission cost.

Theorem 3.5. *The submission cost that maximizes the expected maximal performance,* $\mathbb{E}\left[\max_i X_{\tau_i}\right]$*, is* $\bar{c}$ *if* $\mu \geq 0$ *and is strictly greater than* 0 *if* $\mu < 0$*.*

In this paper's model, the agents who do not submit entries are ineligible for the prize. It is easy to extend our analysis to the modified scenario in which agents who do not submit entries are still eligible (where their values are now their expected values, at equilibrium, given their choice of non-disclosure). Moreover, in some contests, submission costs are refunded in the event of a success; and in some contests, submission costs are refunded in the event of a failure. Those changes are also easy to accommodate.

In the first modification, in which agents who do not submit entries are still eligible, it is obvious that the belief assigned by the principal to non-disclosure must be 0 (since otherwise an agent would benefit by secretly experimenting more in the non-disclosure region). As in the main specification, each agent must put a mass point on 0 but now the size of the mass point γ_N must solve

$$\gamma_N^{n-1} - c = \frac{\gamma_N^{n-1}}{n} \ ;$$

viz., $\gamma_N^{n-1} = nc/(n-1)$. In the second modification, in which only submitters are eligible, but obtain a refund in the event of a victory, the size of the mass point γ_V must solve

$$\gamma_V^{n-1}(1+c) - c = 0 \ ;$$

that is, $\gamma_V^{n-1} = c/(1+c)$. In the third modification, in which only submitters are eligible, but obtain a refund in the event of a loss, the size of the mass point γ_L must solve

$$\gamma_L^{n-1} + (1-\gamma_L^{n-1})c - c = 0 \ ;$$

which reduces to $\gamma_L = 0$. Given these, the unique equilibrium distribution is

$$F(x) = \left(\gamma_\iota + \lambda_\iota \frac{s_B(x)}{s_B(x_0)}\right)^{\left(\frac{1}{n-1}\right)}, \quad \text{on} \quad \left[0, \frac{-\sigma^2}{2\mu}\log\left\{1 - \frac{2\mu}{\sigma^2}\frac{(1-\gamma_\iota)s_B(x_0)}{\lambda_\iota}\right\}\right], \ \iota = N, V, L \ ,$$

where

$$\lambda_\iota := \frac{1 - n\gamma_\iota + (n-1)\gamma_\iota^{\left(\frac{n}{n-1}\right)}}{n}, \ \iota = N, V, L \ .$$

In short, changing the particulars regarding the submission cost is equivalent to changing the cost. Finally, observe that for $c > 0$,

$$\underbrace{0}_{\gamma_L^{n-1}} < \underbrace{\frac{c}{1+c}}_{\gamma_V^{n-1}} < c < \underbrace{\frac{nc}{n-1}}_{\gamma_N^{n-1}},$$

and so the following remark follows from the earlier results.

Remark 3.6. For a fixed c, the average performance of an agent, $\mathbb{E}[X_\tau]$, is highest when non-submitters are also eligible for the prize when $\mu \geq 0$ and is highest when losers' fees are refunded when $\mu < 0$.

It is immediate that the special case of the contest when the drift equals zero and non-submitters are eligible for the prize is equivalent to a competitive persuasion problem in which n agents each privately choose experiments about an idiosyncratic binary state, which they may disclose to a receiver at some cost $c \geq 0$, who then selects the sender whom he esteems highest, breaking ties fairly. The lone modification is that, because agents' values are beliefs, there is an upper bound of 1 on agent's values, so for large enough n or prior value, x_0, agents place a mass point on value 1 as well. Thus, an analog of Theorem 3.5 holds: for all disclosure costs sufficiently small, the principal's (receiver's) welfare is strictly increasing in agents' disclosure costs.

4 Two Exercises

Toward gaining intuition, this section studies two variants of the model. In the first scenario, we replace the submission cost with an entry fee; and in the second, we replace the absorbing boundary with a flow cost of experimentation.

4.1 Entry Costs

What if the submission cost were replaced with an entry cost? That is, suppose that each agent must now pay the cost $c \in [0, \bar{c}]$ "up front," before experimenting; but then, having entered, can disclose his value for free. To mimic the main setting as closely as possible, we impose that each agent's entry decision is private.

In addition, we focus on symmetric equilibria in which each agent participates with probability $\rho \in (0,1]$. Note that $\rho > 0$, since otherwise an agent could deviate profitably by participating and winning the the contest by default. Moreover, each agent's expected payoff from participating is

$$\sum_{k=1}^{n} R_k \binom{n-1}{k-1} H(x)^{n-k} (1 - H(x))^{k-1},$$

where

$$R_k = \sum_{i=k}^{n} \binom{n-1}{i-1} \rho^{n-i} (1-\rho)^{i-1} \quad , \tag{3}$$

which is a prize schedule that satisfies the conditions of NZ. Accordingly, the (symmetric) equilibrium distribution in the experimentation portion of the game is given in Proposition 2.3 of that paper. Moreover, each player's payoff from participating is just the average payoff $\frac{1}{n}\sum_{k=1}^{n} R_k = 1 - \frac{n-1}{n}\rho$. Thus,

Lemma 4.1. *The unique symmetric equilibrium is as follows:*

(i) If the entry cost, c, is sufficiently small ($c \leq \frac{1}{n}$), each agent always enters then chooses the distribution given in Expression 2 with $c = 0$.

(ii) Otherwise ($c > \frac{1}{n}$), each agent enters with probability $\rho = \frac{n}{n-1}(1-c)$ then chooses the distribution specified in Proposition 2.3 of NZ for the prize schedule R_k given in Equation 3.

An immediate corollary of this result is that a low entry cost has no effect on the principal's welfare whatsoever. On the other hand, NZ show that the most inegalitarian prize schedule maximizes experimentation and, therefore, expected performance as well as total performance when the drift is positive. Consequently,

Proposition 4.2. *A low entry cost ($c \leq \frac{1}{n}$) maximizes the (expected) maximal performance, $\mathbb{E}\left[\max_i X_{\tau_i}\right]$ and, if the drift, μ, is positive, maximizes average output $\mathbb{E}[X_\tau]$.*

Proof. From Lemma 4.1, the (effective) prize schedule when $c \leq \frac{1}{n}$ is more inegalitarian (in the sense of the Lorenz order) than the (effective) prize schedule for $c > \frac{1}{n}$. Given this, the second statement of the proposition follows from Corollary 3.5 in NZ. Observing that the maximal performance is bounded by the maximal performance when all n agents enter, the first statement of the proposition follows from Theorem 3.8 in NZ. ∎

In short, when the drift is positive, a large entry cost is doubly bad. It not only reduces experimentation but also results in fewer agents (in expectation) entering the contest in the first place.

4.2 Costly Experimentation

Reassuringly, our main qualitative finding–that submission costs benefit a principal in risk-taking contests–persists in a related (and realistic) environment. Suppose that there are just two agents and that each agent's stochastic process is a martingale: $X_t^i = \sigma B_t^i$,[7] where we have normalized the initial value $x_0 = 0$. No longer is each agent's process absorbed at 0; instead, each agent incurs a flow cost $\gamma > 0$ every instant she runs the process. We maintain the other standing assumptions of the model: agents' observations and stopping decisions are private, an agent must incur a submission cost of $c \geq 0$ to submit her realization, and the winner's (loser's) prize is 1 (0).

As discussed above, because it is only the distribution over values that is payoff-relevant for an agent, we can think of each agent as simply choosing a feasible

[7] The primary reason for this pair of assumptions is tractability: when there are more than two agents or the process has nonzero drift, a closed form for the equilibrium distribution is more difficult to obtain.

distribution, F, from the set of distributions on the real line with mean 0. Now, such a choice is costly: the results of Root (1969) imply that the cost of distribution F is given by the cost functional

$$C(F) := \kappa \int x^2 dF(x) \ ,$$

where $\kappa := \gamma/\sigma^2$. It is straightforward to establish the following result.

Proposition 4.3. *If the cost of entering the contest is large ($c > 1$), agents neither experiment nor submit. If the submission cost is small, the equilibrium distribution is quadratic with one point mass of size c on the lower bound of its support.*

The equilibrium distribution is

$$F(x) = \kappa x^2 + Ax + B + c, \quad \text{on} \quad [-D, E] \ ,$$

where

$$A = \frac{4\sqrt{\kappa}(1-c)^{3/2}}{3}, \quad B = \frac{4}{9}(1-c)^3 \quad D = \frac{2(1-c)^{3/2}}{3\sqrt{\kappa}}, \quad \text{and} \quad E = \frac{(2c+1)\sqrt{1-c}}{3\sqrt{\kappa}} \ .$$

This equilibrium is qualitatively similar to that in our main specification: an agent induces (via some randomization over stopping times) a distribution over stopped values that has a single mass point on the lower bound of its support, after which the agent does not submit; and a continuous portion over values thereafter, which are always submitted.

The (expected) maximal performance, $\mathbb{E}\left[\max_i X_{\tau_i}\right]$, is

$$\frac{2(1-c)^{3/2}(4c+1)}{15\sqrt{\kappa}} \ ,$$

which attains its unique maximum when $c = 1/4$. Thus,

Remark 4.4. Mandating $\min\{\bar{c}, 1/4\} = c$ maximizes maximal performance.

We see that an analog of Theorem 3.5 persists despite the change of environment. A strictly positive submission cost maximizes the maximal performance. Curiously, the optimal c is independent of the variance of the process and the flow cost of experimentation.

5 Discussion

This paper establishes that in contests in which agents compete by risk-taking, introducing submission costs benefits a principal whenever she prefers more experimentation. Thus, in a variety of contests–including those pertaining to innovation, promotion within a firm, investment, status, and persuasion–seemingly inefficient frictions may actually improve a principal's (society's) welfare. Unlike entry fees, which may benefit a principal by increasing effort by dampening competition, submission costs do the opposite and fan competition's flames, encouraging greater risk taking and longer tails in agents' outputs.

One final note: a natural question is to ask what would happen if the contestants could submit the maximal value of the process (rather than the terminal value)?[8] When there is no cost to running the process (and an absorbing boundary at 0) the answer is simple: it is weakly dominant for agents to run the process forever. On the other hand, when there is no such absorbing boundary and agents must instead pay a flow cost of $\gamma > 0$ to run their processes, the problem corresponds to a modified version of Urgun and Yariv (2021)'s setting in which each agent's payoff is now endogenous (an equilibrium object) and the search scope is exogenously fixed at σ. As they note, this problem leads to an intractable ODE, though perhaps a clever guess would yield an equilibrium.

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A Omitted Proofs

A.1 Remark 2.3 Proof

Proof. SS establish uniqueness of the equilibrium with $c = 0$ when there are just two agents in their Proposition 3. Because of this, it suffices to show here that in any equilibrium each agent must submit her entry with the same probability as the other agent. It is clear that the only value that contestants may not submit at equilibrium is 0. Let $a \geq 0$ be the size of the mass agent 1's distribution places on 0 and $b \geq 0$ be the size of the mass agent 2's distribution places on 0. We must have $a, b \geq c$; otherwise, if say $a < c$, agent 2 would strictly prefer to deviate and not submit a positive measure of values that lie strictly above 0. On the other hand, if $a > c$, agent 2 prefers to deviate and always submit. Thus $a = b = c$. ■

A.2 Lemma 3.1 Proof

Proof. We prove this result for the general case in which agents' processes are non-negative time homogeneous diffusions. The equilibrium distribution is

$$G(x) = \left(c + \lambda \frac{s(x)}{s(x_0)}\right)^{\left(\frac{1}{n-1}\right)}, \quad \text{on} \quad \underbrace{\left[0, \psi\left((1-c)\frac{s(x_0)}{\lambda}\right)\right]}_{=:[0,\bar{x}(c)]},$$

where $s(x)$ is the scale function of X and $\psi(\cdot) := s^{-1}(\cdot)$.

Directly, $G(0) = c^{\left(\frac{1}{n-1}\right)}$, which is obviously strictly increasing in c. Likewise,

$$\frac{\partial}{\partial c}\left\{\psi\left((1-c)\frac{s(x_0)}{\lambda}\right)\right\} = \psi'\left((1-c)\frac{s(x_0)}{\lambda}\right)s(x_0)\frac{\partial}{\partial c}\left\{\frac{1-c}{\lambda}\right\} > 0\,,$$

since s is strictly monotone. As a result, by the intermediate value theorem, there exists at least one point x in the interior of $\operatorname{supp} G \cup \operatorname{supp} \hat{G}$ where $G(x) = \hat{G}(x)$.

Next, we need to show that this intersection point is unique. Define $\Upsilon(x) := G(x) - \hat{G}(x)$, which can be written out as

$$\Upsilon(x) = \left(c + \lambda \frac{s(x)}{s(x_0)}\right)^{\left(\frac{1}{n-1}\right)} - \left(\hat{c} + \hat{\lambda} \frac{s(x)}{s(x_0)}\right)^{\left(\frac{1}{n-1}\right)},$$

where $\hat{\lambda}$ is defined in the obvious way. Directly, $\Upsilon'(x)$ has the same sign as

$$\underbrace{\lambda\left(c + \lambda \frac{s(x)}{s(x_0)}\right)^{\left(\frac{1}{n-1}-1\right)}}_{=:r(\lambda)} - \hat{\lambda}\left(\hat{c} + \hat{\lambda} \frac{s(x)}{s(x_0)}\right)^{\left(\frac{1}{n-1}-1\right)}.$$

If $n = 2$, this reduces to $\lambda - \hat{\lambda} < 0$, as required. For the remainder let $n \geq 3$. Then,

$$\begin{aligned} r'(\lambda) &= \left(c + \lambda \frac{s(x)}{s(x_0)}\right)^{\left(\frac{1}{n-1}-1\right)} + \lambda \frac{s(x)}{s(x_0)}\left(\frac{1}{n-1} - 1\right)\left(c + \lambda \frac{s(x)}{s(x_0)}\right)^{\left(\frac{1}{n-1}-2\right)} \\ &= \left(c + \lambda \frac{s(x)}{s(x_0)}\frac{1}{n-1}\right)\left(c + \lambda \frac{s(x)}{s(x_0)}\right)^{\left(\frac{1}{n-1}-2\right)} > 0 \end{aligned}.$$

Now, suppose for the sake of contradiction that $\Upsilon'(x) \geq 0$, which implies that

$$\hat{\lambda}\left(c+\hat{\lambda}\frac{s(x)}{s(x_0)}\right)^{\left(\frac{1}{n-1}-1\right)} - \hat{\lambda}\left(\hat{c}+\hat{\lambda}\frac{s(x)}{s(x_0)}\right)^{\left(\frac{1}{n-1}-1\right)} \geq 0 \ ,$$

or

$$\left(c+\hat{\lambda}\frac{s(x)}{s(x_0)}\right)^{-\left(\frac{n-2}{n-1}\right)} \geq \left(\hat{c}+\hat{\lambda}\frac{s(x)}{s(x_0)}\right)^{-\left(\frac{n-2}{n-1}\right)} \ ,$$

a contradiction because $c > \hat{c}$. ■

A.3 Theorem 3.2 Proof

Proof. As we do for the previous lemma, we establish this result for the general case. We follow the proof of Theorem 3.2 in NZ virtually verbatim.

Via integration by parts,

$$\mathbb{E}_{\hat{G}}[\phi(x)] - \mathbb{E}_{G}[\phi(x)] = -\int_0^{\bar{x}(c)} \left(\hat{G}(x) - G(x)\right)\phi'(x)\,dx \ .$$

By Lemma 3.1, $\hat{G}$ is strictly single-crossing with regard to G with some intersection point $x^* \in (0, \bar{x}(c))$. Thus,

$$\int_0^{\bar{x}(c)} \left(\hat{G}(x) - G(x)\right)s'(x)\frac{\phi'(x)}{s'(x)}dx \geq \frac{\phi'(x^*)}{s'(x^*)}\int_0^{\bar{x}(c)} \left(\hat{G}(x) - G(x)\right)s'(x)\,dx \ .$$

Next, we again integrate by parts

$$\int_0^{\bar{x}(c)} \left(\hat{G}(x) - G(x)\right)s'(x)\,dx = \left(\hat{G}(x) - G(x)\right)s(x)\Big|_0^{\bar{x}(c)} - \int_0^{\bar{x}(c)} (\hat{g}(x) - g(x))\,s(x)\,dx = 0 \ ,$$

since by definition the feasible distributions, G, are those that satisfy

$$\int_{\mathbb{R}_+} \frac{s(x)}{s(x_0)}dG(x) = 1 \ .$$

Combining expressions yields part 1 of the theorem, and the second part can be obtained in virtually identical fashion *mutatis mutandis*. ■

A.4 Theorem 3.5 Proof

Proof. We keep the convention that $c > \hat{c} \geq 0$ with hats over the corresponding objects to the latter. By Lemma 3.1 $\hat{F}$ is strictly single crossing with regard to F and so F^{-1} is strictly single crossing with regard to $\hat{F}^{-1}$. Let y_* be the unique crossing point. First, let $\mu \geq 0$, in which case we may follow the proof of Theorem 3.8 in NZ. We have

$$\int_{c\left(\frac{1}{n-1}\right)}^{1} y^{n-1} F^{-1}(y)\,dy - \int_{\hat{c}\left(\frac{1}{n-1}\right)}^{1} y^{n-1} \hat{F}^{-1}(y)\,dy > y_*^{n-1}\left[\int_{c\left(\frac{1}{n-1}\right)}^{1} F^{-1}(y)\,dy - \int_{\hat{c}\left(\frac{1}{n-1}\right)}^{1} \hat{F}^{-1}(y)\,dy\right] \geq 0\,,$$

where the last inequality follows from Corollary 3.4 since $\mu \geq 0$.

Now let $\mu < 0$, and observe that it suffices to show that $\mathbb{E}[\max_i X_i]$ is strictly increasing in c at $c = 0$. Via Leibniz's rule, we have

$$\frac{d}{dc}\left\{\int_{c\left(\frac{1}{n-1}\right)}^{1} y^{n-1} F^{-1}(y)\,dy\right\} = \int_{c\left(\frac{1}{n-1}\right)}^{1} \frac{\partial}{\partial c}\left\{y^{n-1} F^{-1}(y)\right\} dy\,. \tag{A1}$$

Directly,

$$F^{-1}(y) = -\alpha \ln\left\{1 - \frac{\left(y^{n-1} - c\right) s(x_0)}{\lambda \alpha}\right\},$$

where $\alpha := \sigma^2/(2\mu)$. Using this, Expression A1 evaluated at 0 has the same sign as

$$\int_0^1 \frac{n y^{n-1} - 1}{1 + n\beta y^{n-1}} y^{n-1}\,dy > \frac{\frac{1}{n}}{1+\beta} \int_0^1 \left(n y^{n-1} - 1\right) dy = 0\,,$$

where $\beta := -(1 - \exp\{-x_0/\alpha\}) > 0$. ∎

A.5 Proposition 4.3 Proof

Proof. It is easy to see that when $c > 1$, not experimenting then not submitting is strictly dominant. If $c \leq 1$, the logic is analogous to that of Proposition 2.2, with one exception: as noted in Lemma 6 of Seel and Strack (2016), we also need the right-hand side derivative of the equilibrium distribution to be 0 at the lower bound of its support, $-D$. The equilibrium distribution must leave the other contestant willing to randomize; equivalently, the equilibrium distribution must be such that a contestant's payoff for any value x in the support of her equilibrium distribution must lie on a line. *Viz.*, we must have

$$Ax + B = F(x) - kx^2 - c \ ,$$

for all x in the support of the equilibrium distribution. Rearranging this, the equilibrium cdf is

$$F(x) = Ax + B + kx^2 + c, \quad \text{on} \quad [-D, E] \ , \qquad \text{(linearity of payoff)}$$

where variables A, B, D, and E comprise the unique solution to the following system of four equations such that F is feasible:

$$c = F(-D) = -AD + B + kD^2 + c \ , \qquad \text{(support condition I)}$$

$$1 = F(E) = AE + B + kE^2 + c \ , \qquad \text{(support condition II)}$$

$$\int_{-D}^{E} (A + 2\kappa x)\, x dx = Dc \ , \qquad \text{(feasibility)}$$

$$A = 2\kappa D \ . \qquad \text{(derivative condition)}$$

This yields the values given in the text. ■