Attraction versus Persuasion: Information Provision in Search Markets

Supplementary Appendix (For Online Publication)

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Contents

A Two Examples from Section 3.3

This section contains two examples that illustrate that the set of effective-value distributions that can be induced deterministically is not convex, and that not all Bayes-plausible distributions (with mean $\underaccent{\bar}{U} = \mu - c$) are inducible, respectively. First, the non-convexity example:

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Example A.1. Consider the effective-value distribution supported on $\left\{0, \underline{U}, \bar{U}\right\}$ with respective probabilities $(1-\mu)/2$, $1/2$ and $\mu/2$. It can be induced by a mixed strategy that randomizes fairly between full information and no information. If it could be generated by some deterministic distribution over posteriors F_i, then this F_i must have reservation value $\bar{U}$ (by Equation (2) in the text). However, by Equation (3) in the text, its expected effective value conditional on falling short of $\bar{U}$ must be zero, contradicting that F_i assigns positive weight to $\underline{U} > 0$. We have identified an effective-value distribution that can be induced only via mixing.

Second, the inducibility example:

Example A.2. Consider an effective-value distribution supported on $\left\{\underline{U}/2, \bar{U}\right\}$ with respective probabilities $2(1-\mu)/(2-\mu)$ and $\mu/(2-\mu)$ (so that its mean is $\underline{U}$). We can invoke Lemma 3.5 to show that it is not inducible. If it were, its associated reservation-value distribution would have an atom at $\bar{U}$. The corresponding distribution over posteriors $F_i\left(\cdot;\bar{U}\right)$ must have an atom at effective value 0. Equation (3) in the text then implies an atom at 0, a contradiction.

B An Asymmetric Equilibrium

We begin this section with the following proposition, which points out that although asymmetric equilibria are impossible when there are just two firms, they may exist when there are three or more firms.

Proposition B.1. *If $n = 2$, there exist no asymmetric equilibria. If $n \geq 3$ then for any $\mu < \bar{\mu}$ that is sufficiently close to $\bar{\mu}$, there exists an equilibrium in which $n-1$ firms choose the binary distribution with support $\{0, 1 - c/\mu\}$ and one firm chooses the uniform distribution on $[0, 2(\mu - c)]$.*

The precise condition required for this equilibrium to exist is

$$\frac{2(1-\mu)}{n+1-2\mu} \geq (1-\mu)^{n-1} \geq \frac{1}{n}\,. \tag{S1}$$

Proof. Because the game is zero-sum, the first part of this result is trivial. If there existed an asymmetric equilibrium then there would exist multiple pure strategy equilibria, which we know is false.

To establish the second part of the proposition, observe that the corresponding distribution over values played by the $n-1$ firms is just the prior and is induced by providing full information. For each of these firms, its payoff as a function of the induced effective value, w, is

$$\Pi(w) = \begin{cases} \frac{(1-\mu)^{n-2}}{2(\mu-c)} w, & \text{if} \quad w \in [0, 2(\mu - c)] \\ (1-\mu)^{n-2}, & \text{if} \quad w \in \left(2(\mu-c), \bar{U}\right) \\ \frac{1-(1-\mu)^{n-1}}{\mu(n-1)}, & \text{if} \quad w = \bar{U} \end{cases} .$$

Thus, the optimal distribution is either the prior (full information), yielding a payoff of $\left(1-(1-\mu)^{n-1}\right)/(n-1)$; or has support on $[0, 2(\mu - c)]$, yielding a payoff of $(1-\mu)^{n-2}/2$. Hence, we need

$$\frac{1-(1-\mu)^{n-1}}{n-1} \geq \frac{(1-\mu)^{n-2}}{2} \quad \Longleftrightarrow \quad \frac{2(1-\mu)}{n+1-2\mu} \geq (1-\mu)^{n-1} .$$

The construction of the effective-value distribution for the firm that is choosing the uniform distribution is described in the paper (it is just the low mean two firm distribution and can be done, e.g., by mixing over binary distributions over posteriors). From any distribution over effective values other than that corresponding to full information (the prior), firm n's payoff is $(1-\mu)^{n-1}$; whereas its payoff from full information is $1/n$, since the vector of strategies would then be symmetric. Thus, the optimal distribution is either the prior, yielding a payoff of $1/n$; or any distribution with support on $[0, 1 - c/\mu]$, yielding a payoff of $(1-\mu)^{n-1}$. Accordingly, we need $(1-\mu)^{n-1} \geq 1/n$. Both conditions combine to yield Expression ($S1$). ■

In this equilibrium, $n-1$ firms provide full information, and the nth firm chooses a distribution over effective values that is identical to the equilibrium distribution when there are two firms and μ is low. Surprisingly, this equilibrium yields a higher consumer welfare than the symmetric equilibrium.[1] This proposition contrasts nicely with the results of Armstrong et al. (2009), who show that when firms are symmetric, making a firm prominent lowers consumer welfare. Here, we encounter an equilibrium in which $n-1$ firms are endogenously prominent, yet consumer welfare rises despite the asymmetric behavior. This is because, in contrast to Armstrong et al. (2009), in which the non-prominent firms raise prices to the detriment of the consumer, it does not matter to the consumer here what the last firm does.

While this proposition does not exhaustively identify all asymmetric equilibria, it suggests that an interesting avenue for future research could be comparing the properties of asymmetric equilibria with the symmetric equilibria on which this paper focuses. If $n \geq 3$, there also exist asymmetric equilibria for certain parameter ranges in which each firm chooses a pure strategy. In the next subsection we show

[1]The reason is as follows. First, because $\mu < \bar{\mu}$ the symmetric equilibrium does not involve full disclosure. Moreover, as the search cost vanishes, the consumer's welfare in the asymmetric equilibrium identified in the proposition converges to the first-best.

that any such equilibrium must be one in which all but one firm provide full information (and induce the maximum reservation value).

B.1 Asymmetric Pure-Strategy Equilibria

Proposition B.2. *In any asymmetric equilibrium in which each firm's choice of signal is deterministic, $n-1$ firms provide full information.*

Proof. Let us assume that there exists such an equilibrium. Let U_{max} denote the maximal reservation that is induced on path (obviously $U_{max} \leq \bar{U}$). First, suppose for the sake of contradiction that $U_{max} < \bar{U}$. There are two subcases: either multiple firms induce U_{max} or only one does. However, the technique of effective-value optimization that we introduce in the paper immediately implies that in both cases, a firm has a profitable deviation. Indeed, if multiple firms induce U_{max} in this purported equilibrium, one of those firms can deviate profitably by providing slightly more information, thereby inducing some reservation value $U > U_{max}$. On the other hand, if only one firm induces U_{max}, although its payoff from the realized expected value U_{max} is 1, so is its payoff from any realized expected value $U \in (U_{max} - \varepsilon, U_{max}]$ for all $\varepsilon > 0$ sufficiently small. Consequently, this firm can deviate by providing *less* information. We may conclude that $U_{max} = \bar{U}$.

Second, suppose that $U_{max} = \bar{U}$. Note that for any asymmetric equilibrium to exist, there must be three or more firms. Define $\hat{U} \equiv \max\left\{U : \; U \text{ induced on path}, U < \bar{U}\right\}$, which is well-defined since there exist only finitely many firms and we have imposed that the equilibrium under examination is asymmetric. There are two subcases. Either each of the other firms is inducing $\bar{U}$ or at least one is not. The analysis for the latter case is identical to that of the preceding paragraph: if multiple firms are inducing $\hat{U}$, one can deviate by providing more information; if only one is, it can deviate by providing less information. Thus, $n-1$ firms induce $\bar{U}$ and one induces $\hat{U}$. ■

C Equilibrium Characterization with Heterogeneous Firms

What happens when there are two firms with different expected qualities? Without loss of generality, let $\mu_1 \geq \mu_2$. We find that there are four different regions of the parameter space, each of which begets a different variety of equilibrium. First, if the gap between the means is large enough–specifically, if the maximum reservation value that firm 2 can induce is weakly less than the minimum reservation value that firm 1 can induce–then in all equilibria, firm 1 chooses the degenerate distribution over effective values (corresponding to no information) and firm 2 chooses any distribution over effective values. The consumer visits firm 1 first and selects it for sure.

Next, if the gap between means is not as large and firm 2's mean is not too high ($\mu_2 < 1/2$), there are two regions in which both firms' payoff functions–and hence both firms' distributions over effective values–have the familiar linear structure. In both of these regions, firm 2 places an atom on effective value 0, and in one of the regions firm 1 places an atom on firm 2's maximum effective value ($1 - c/\mu_2$).

Finally, if the gap in means is not large but $\mu_2 \geq 1/2$, firm 1 chooses a binary distribution over effective values supported on 0 and $1 - c/\mu_2$, and firm 2 chooses a linear distribution over effective values. This is a similar equilibrium, qualitatively, to the asymmetric equilibrium from the previous subsection. The attraction incentive dominates for firm 1 who is always visited first. Firm 2, on the other hand, is content to "pick up the scraps." It is visited second but always selected by the consumer if visited. Moreover, this equilibrium also shares the same property as its analog when firms are homogeneous. The consumer's payoff converges to the first-best (full information) as the search cost vanishes. Thus, our result from the homogeneous firms setting–that search frictions beget the first-best level of information provided the average quality is sufficiently high–carries over to the heterogeneous firms setting. The following theorem provides a synopsis of these results.

Theorem C.1. *Let the average match value for firm* 1 *be weakly greater than the average match value for firm* 2*:* $\mu_1 \geq \mu_2$*. Then,*

(i) If $\mu_1 - c \geq 1 - c/\mu_2$*, there is a collection of equilibria in which firm* 1 *chooses the degenerate distribution over effective values with support* $\{\mu_1 - c\}$ *and firm* 2 *chooses any distribution over effective values.*

(ii) If $\mu_2 \leq 1/2$ *and* $1 - c/\mu_2 \geq 2(\mu_1 - c)$*, there is an equilibrium in which firm* 1 *and firm* 2 *choose linear distributions over effective values. Firm* 2 *places a mass point on the effective value* 0*.*

(iii) If $\mu_2 \leq 1/2$ *and* $2(\mu_1 - c) \geq 1 - c/\mu_2 \geq \mu_1 - c$*, there is an equilibrium in which firm* 1 *and firm* 2 *choose linear distributions over effective values. Firm* 2 *places a mass point on the effective value* 0*, whereas firm* 1 *places a mass point on the effective value* $1 - c/\mu_2$*.*

(iv) If $\mu_2 \geq 1/2$ *and* $\mu_1 - c < 1 - c/\mu_2$*, there is an equilibrium in which firm* 1 *chooses the binary distribution over effective values with support* $\{0, 1 - c/\mu_2\}$ *and firm* 2 *chooses a distribution over effective values that is piece-wise linear with one discontinuity.*

The theorem compiles the results from the following four lemmas. One-by-one,

Lemma C.2. *If* $\mu_1 - c \geq 1 - c/\mu_2$*, there is a collection of equilibria in which firm* 1 *chooses the degenerate distribution over effective values,* $\mu_1 - c$ *with probability* 1*, and firm* 2 *chooses any distribution over effective* *values. Firm 1's distribution over effective values corresponds, e.g., to a completely uninformative signal.*

Proof. The result is trivial. Firm 1 is visited first and selected with certainty. ■

Lemma C.3. *If $\mu_2 \le 1/2$ and $1 - c/\mu_2 \ge 2(\mu_1 - c)$, there is an equilibrium in which firm 1 and firm 2 choose distributions over effective values $H_1(w)$ and $H_2(w)$, respectively, where*

$$H_1(w) = \frac{w}{2(\mu_1 - c)}, \quad \text{on} \quad [0, 2(\mu_1 - c)],$$

and

$$H_2(w) = 1 - \frac{\mu_2 - c}{\mu_1 - c} + \left(\frac{\mu_2 - c}{\mu_1 - c}\right)\frac{w}{2(\mu_1 - c)}, \quad \text{on} \quad [0, 2(\mu_1 - c)] \ .$$

Proof. It is easy to verify that these distributions are feasible, but it remains to verify that they are inducible. To that end, we construct them as follows. Firm 1's random (reservation) value U_1 is distributed according to distribution G_1:

$$G_1(u) \equiv \mathbb{P}(U_1 \le u) = \frac{1}{\mu_1 - c}u - 1, \quad \text{on} \quad [\mu_1 - c, 2(\mu_1 - c)] \ ,$$

where for each $u \in [\mu_1 - c, 2(\mu_1 - c)]$, the distribution over posteriors, $F(\cdot; u)$, is the binary distribution with support $\{2(\mu_1 - c) - u, u + 2c\}$. In turn, firm 2's random (reservation) value U_2 is distributed according to distribution G_2.

$$G_2(u) \equiv \mathbb{P}(U_2 \le u) = \frac{1}{\mu_1 - c}u - 1, \quad \text{on} \quad [\mu_1 - c, 2(\mu_1 - c)] \ ,$$

where for each $u \in [\mu_1 - c, 2(\mu_1 - c)]$, $F(\cdot; u)$ is a *ternary* distribution with pmf

$$P(\cdot; u) = \begin{Bmatrix} 0 & b(u) & a(u) \\ 1 - \frac{\mu_2 - c}{\mu_1 - c} & \frac{\mu_2 - c}{2(\mu_1 - c)} & \frac{\mu_2 - c}{2(\mu_1 - c)} \end{Bmatrix}, \quad \text{where} \quad a(u) \equiv u + 2c\frac{\mu_1 - c}{\mu_2 - c} \quad \text{and} \quad b(u) \equiv 2(\mu_1 - c) - u \ .$$

The top row of the matrix is the support of the distribution and the bottom row the associated probability weights. Evidently, these constructions yield the desired distributions over effective values. ■

Lemma C.4. *If $\mu_2 \le 1/2$ and $2(\mu_1 - c) > 1 - c/\mu_2 > \mu_1 - c$, there is an equilibrium in which firm 1 and firm 2 choose distributions over effective values H_1 and H_2, respectively, where*

$$H_1(w) = 2\left(1 - \frac{\mu_2(\mu_1 - c)}{\mu_2 - c}\right)\frac{\mu_2}{\mu_2 - c}w, \quad \text{on} \quad \left[0, \frac{\mu_2 - c}{\mu_2}\right] \ ,$$

and

$$H_2(w) = 1 - 2\mu_2 + 2\mu_2\frac{\mu_2}{\mu_2 - c}w, \quad \text{on} \quad \left[0, \frac{\mu_2 - c}{\mu_2}\right] \ .$$

Proof. As above, it is easy to verify that these distributions are feasible, but it remains to verify that they are inducible. To that end, we construct them as follows.

Firm 1's random (reservation) value U_1, is distributed according to distribution G_1:

$$G_1(u) \equiv \mathbb{P}(U_1 \leq u) = 4\frac{\mu_2}{\mu_2 - c}\left(1 - \frac{\mu_2(\mu_1 - c)}{\mu_2 - c}\right)(u - (\mu_1 - c)), \quad \text{on} \quad \left[\mu_1 - c, \frac{\mu_2 - c}{\mu_2}\right],$$

where for each $u \in [\mu_1 - c, 1 - c/\mu_2)$, $F(\cdot; u)$ is binary with support $\{b(u), a(u)\}$, where $a(u) \equiv u + 2c$ and $b(u) \equiv 2(\mu_1 - c) - u$; and $F_{\frac{\mu_2 - c}{\mu_2}}(x)$ is defined as

$$F_{\frac{\mu_2-c}{\mu_2}}(x) = \begin{cases} \frac{2\left(\mu_1 - c - \frac{\mu_2 - c}{\mu_2}\right)}{3\left(\frac{\mu_2-c}{\mu_2}\right)^2 - 8(\mu_1 - c)\frac{\mu_2-c}{\mu_2} + 4(\mu_1-c)^2} x, & \text{if} \quad x \in \left[0, 2(\mu_1 - c) - \frac{\mu_2 - c}{\mu_2}\right] \\ 1 - \left(\frac{\mu_2 - c}{\mu_2} + 3c - \frac{2(\mu_1 - c)\mu_2}{\mu_2 - c} c\right), & \text{if} \quad x \in \left[2(\mu_1 - c) - \frac{\mu_2-c}{\mu_2}, \frac{\mu_2 - c}{\mu_2} + 3c - \frac{2(\mu_1-c)\mu_2}{\mu_2 - c} c\right) \\ 1, & \text{if} \quad x \in \left[\frac{\mu_2 - c}{\mu_2} + 3c - \frac{2(\mu_1 - c)\mu_2}{\mu_2 - c} c, 1\right] \end{cases}.$$

Viz., $F_{\frac{\mu_2-c}{\mu_2}}(x)$ has a point mass of size

$$\frac{\frac{\mu_2 - c}{\mu_2}}{3\frac{\mu_2 - c}{\mu_2} - 2(\mu_1 - c)} \quad \text{on} \quad \frac{\mu_2 - c}{\mu_2} + 3c - \frac{2(\mu_1 - c)\mu_2}{\mu_2 - c} c\,.$$

Evidently, a is increasing in u and takes values in the interval $[\mu_1 + c, 1 - c/\mu_2 + 2c]$; and b is decreasing in u and takes values in the interval $[\mu_1 - c, 2(\mu_1 - c) - 1 + c/\mu_2]$. We should verify four things:

Claim C.5. *The upper bound of $a(u)$ is less than 1, i.e., $1 - c/\mu_2 + 2c \leq 1$.*

Proof. Directly,

$$\frac{\mu_2 - c}{\mu_2} + 2c = 1 - \frac{c}{\mu_2} + 2c \leq 1 - 2c + 2c = 1\,,$$

since $\mu_2 \leq 1/2$. ■

Claim C.6. $3c - 2(\mu_1 - c)\mu_2 c/(\mu_2 - c) \leq 2c$.

Proof. This holds if and only if

$$1 \leq \frac{2(\mu_1 - c)\mu_2}{\mu_2 - c} \quad \Longleftrightarrow \quad \frac{\mu_2 - c}{\mu_2} \leq 2(\mu_1 - c)\,,$$

which holds by assumption. ■

Claim C.7. *$F_{\frac{\mu_2-c}{\mu_2}}(x)$ does not have support above 1, i.e., $1 - c/\mu_2 + 3c - 2(\mu_1 - c)\mu_2 c/(\mu_2 - c) \leq 1$.*

Proof. Directly,

$$\frac{\mu_2 - c}{\mu_2} + 3c - \frac{2(\mu_1 - c)\mu_2}{\mu_2 - c} c \leq \frac{\mu_2 - c}{\mu_2} + 2c \leq 1\,,$$

where the first inequality follows from Claim C.6, and the second inequality from Claim C.5. ■

Claim C.8.

$$\frac{\mu_2 - c}{\mu_2} + 3c - \frac{2(\mu_1 - c)\mu_2}{\mu_2 - c} c > \frac{\mu_2 - c}{\mu_2} \geq 2(\mu_1 - c) - \frac{\mu_2 - c}{\mu_2} .$$

Proof. The right hand inequality holds since $1 - \mu_2/c \geq \mu_1 - c$. Now the left-hand inequality:

$$\frac{\mu_2 - c}{\mu_2} + 3c - \frac{2(\mu_1 - c)\mu_2}{\mu_2 - c} c \geq \frac{\mu_2 - c}{\mu_2} + \frac{3(\mu_1 - c)\mu_2}{\mu_2 - c} c - \frac{2(\mu_1 - c)\mu_2}{\mu_2 - c} c > \frac{\mu_2 - c}{\mu_2} ,$$

since $1 - c/\mu_2 \geq \mu_1 - c$. ∎

Note that for the special sub-case where $2(\mu_1 - c) - (\mu_2 - c) \geq 1 - c/\mu_2 > \mu_1 - c$, the distribution over effective values can also be generated by a pure strategy distribution over values F^*, where

$$F^*(x) = \begin{cases} 2\frac{\mu_2}{\mu_2 - c}\left(1 - \frac{(\mu_1 - c)\mu_2}{\mu_2 - c}\right)x, & \text{if} \quad x \in \left[0, \frac{\mu_2 - c}{\mu_2}\right] \\ 2\left(1 - \frac{(\mu_1 - c)\mu_2}{\mu_2 - c}\right), & \text{if} \quad x \in \left[\frac{\mu_2 - c}{\mu_2}, \frac{\mu_2 - c}{\mu_2} + \frac{c(\mu_2 - c)}{2(\mu_1 - c)\mu_2 - (\mu_2 - c)}\right) \\ 1, & \text{if} \quad x \in \left[\frac{\mu_2 - c}{\mu_2} + \frac{c(\mu_2 - c)}{2(\mu_1 - c)\mu_2 - (\mu_2 - c)}, 1\right] \end{cases} .$$

It suffices to check

$$\frac{\mu_2 - c}{\mu_2} + \frac{c(\mu_2 - c)}{2(\mu_1 - c)\mu_2 - (\mu_2 - c)} \leq 1 \quad \Longleftrightarrow \quad 2(\mu_1 - c) - (\mu_2 - c) \geq 1 - \frac{c}{\mu_2} .$$

Firm 2's random (reservation) value U_2, is distributed according to distribution G_2:

$$G_2(u) \equiv \mathbb{P}(U_2 \leq u) = \frac{2\mu_2}{\mu_2 - c} u - 1, \quad \text{on} \quad \left[\frac{\mu_2 - c}{2\mu_2}, \frac{\mu_2 - c}{\mu_2}\right] ,$$

where for each $u \in [1/2 - c/(2\mu_2), 1 - c/\mu_2]$, $F(\cdot; u)$ is given by the *ternary* distribution with pmf

$$P(\cdot; u) = \begin{Bmatrix} 0 & b(u) & a(u) \\ 1 - 2\mu_2 & \mu_2 & \mu_2 \end{Bmatrix}, \quad \text{where} \quad a(u) \equiv u + \frac{c}{\mu_2} \quad \text{and} \quad b(u) \equiv \frac{\mu_2 - c}{\mu_2} - u .$$

∎

Lemma C.9. *If $\mu_2 \geq 1/2$ and $1 - c/\mu_2 \geq \mu_1 - c$, there is an equilibrium in which firm 1 chooses the binary distribution over effective values with support $\{0, 1 - c/\mu_2\}$, and firm 2 chooses the distribution over effective values $H_2(w)$, where*

$$H_2(w) = \begin{cases} \frac{(1-\mu_2)^2}{\mu_2}\left(\frac{w}{\mu_2 - c}\right), & \textit{if} \quad w \in [0, \mu_2 - c) \\ \frac{\mu_2}{\mu_2 - c} w, & \textit{if} \quad w \in \left[\mu_2 - c, 1 - \frac{c}{\mu_2}\right] \\ 1, & \textit{if} \quad w \geq 1 - \frac{c}{\mu_2} \end{cases} .$$

Proof. Firm 1 induces its effective-value distribution by choosing the binary distribution over posteriors with support $\{0, \mu_1 (\mu_2 - c) / (\mu_2 (\mu_1 - c))\}$, and firm 2 induces its effective-value distribution by choosing distribution G_2 over reservation values:

$$G_2(u) \equiv \mathbb{P}(U_2 \leq u) = \frac{u}{\mu_2 - c} - \frac{1-\mu_2}{\mu_2} \quad \text{on} \quad \left[\mu_2 - c, \frac{\mu_2 - c}{\mu_2}\right],$$

where for each $u \in [\mu_2 - c, 1 - c/\mu_2]$, $F(\cdot; u)$ is binary with support $\{(\mu_2 - c - \mu_2 u) / (1-\mu_2), u + c/\mu_2\}$. ∎

D Relevant Outside Option

While our main analysis has abstracted away the consumer's outside option, the tools we developed can be applied to the setting in which the consumer has a relevant outside option. Suppose the consumer has an outside option $u_0 \in (0, \bar{U})$ to which she may always return upon quitting her search.[2] A possible interpretation of the outside option is a common product price that is exogenously determined. With a binding outside option, the game between the firms is no longer zero-sum, as the consumer will refrain from making any purchase if the firms' quality realizations turn out to be less than u_0, an event that we find has a strictly positive probability. Following Section 4 in the text, we focus on a symmetric equilibrium that gives rise to a payoff function that is linear on the interior of its support, analogous to that reported in Lemma 4.1. A minor difference here is that the payoff function must be flat at 0 in the interval $[0, u_0)$, which necessitates a discrete jump at u_0. Consequently, the effective-value distribution must possess an atom at 0 (even when μ is low).

Following Section 4, denote by H_i a symmetric-equilibrium distribution of effective values chosen by each firm, let $\alpha \in [0,1]$ be the probability that a firm offers full information, and let $\hat{U} \equiv \sup\left(\text{supp}(H_i) / \left\{\bar{U}\right\}\right)$. With a relevant outside option, we say a payoff function has the linear structure if it takes one of the following forms.

Semi-linear form:

$$\Pi(w; H_i) = \begin{cases} 0, & \text{if} \quad w = [0, u_0) \\ (\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0} \times (w - u_0), & \text{if} \quad w \in \left[u_0, \hat{U}\right] \\ (1-\alpha\mu)^{n-1}, & \text{if} \quad w \in (\hat{U}, \bar{U}) \\ \frac{1-(1-\alpha\mu)^n}{n\alpha\mu}, & \text{if} \quad w = \bar{U} \end{cases}, \tag{S2}$$

where $(\alpha(1-\mu))^{n-1} / u_0 > \left[(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}\right] / \left(\hat{U} - u_0\right)$. Note that this form includes the full-disclosure equilibrium as a special case, with $\alpha = 1$ and $\hat{U} = 0$.

[2]The condition $u_0 < \bar{U}$ ensures that search is not strictly dominated for the consumer and thus remains relevant.

Fully-linear form:

$$\Pi(w; H_i) = \begin{cases} 0, & \text{if} \quad w = [0, u_0) \\ (1-\alpha\mu)^{n-1} \times \frac{w}{\hat{U}}, & \text{if} \quad w \in \left[u_0, \hat{U}\right] \\ (1-\alpha\mu)^{n-1}, & \text{if} \quad w = \left(\hat{U}, \bar{U}\right) \\ 1, & \text{if} \quad w = \bar{U} \end{cases} . \tag{S3}$$

In the rest of this section, we prove the following result, which is also stated in the text.

Proposition D.1. *Suppose the consumer's outside option is relevant, i.e.,* $u_0 \in \left(0, \bar{U}\right)$, *and that* $n \geq 3$. *A symmetric equilibrium that begets a linear payoff function exists and is unique (up to the effective-value distribution). There is a cutoff* $\mu^{FD} \in (0, 1)$ *such that the equilibrium has full disclosure whenever* $\mu \geq \mu^{FD}$. *If the equilibrium involves partial disclosure, firms must mix.*

It is clear that if the payoff function takes the linear form described in Expressions (S2) and (S3), it is a best response to offer an effective-value distribution supported on $\left[u_0, \hat{U}\right] \cup \left\{0, \bar{U}\right\}$ in the former case and $\left[u_0, \hat{U}\right] \cup \{0\}$ in the latter case. The lemma below uses the necessary conditions for a symmetric equilibrium–in particular, the dependence of the equilibrium payoff on the atom assigned to effective value 0–to pin down the specific equilibrium form (whether it is semi-linear, fully linear or full disclosure; as well as the feasible values of α and $\hat{U}$) for each combination of the average quality μ and the outside option u_0.

Lemma D.2. *Suppose* $n \geq 3$, *and let* μ^{FD} *be the unique solution to equation* $1 - (1-\mu)^n = n(1-\mu)^{n-1}$. *Within the class of symmetric equilibria that beget a linear payoff function, the unique form of the equilibrium depends on the average quality* μ *and the outside option* u_0 *as follows.*

(i) If $\mu \geq \mu^{FD}$, *the equilibrium has full disclosure for all* $u_0 > 0$.

(ii) For each $\mu \in \left(1/n, \mu^{FD}\right)$, *there are cutoffs* u_0^L *and* u_0^{FD} *such that the equilibrium payoff function necessarily takes the semi-linear form if* $u_0 < u_0^L$, *takes the fully-linear form if* $u_0 \in [u_0^L, u_0^{FD})$, *and has full disclosure if* $u_0 \geq u_0^{FD}$.

(iii) For each $\mu \leq 1/n$, *there is a cutoff* u_0^{FD} *such that the equilibrium payoff function necessarily takes the fully-linear form if* $u_0 < u_0^{FD}$ *and has full disclosure if* $u_0 \geq u_0^{FD}$.

Proof. Consider first the case of full disclosure in equilibrium. If all other firms are fully revealing, the expected payoff of a firm by following suit is $(1-(1-\mu)^n)/n$. The optimal deviation is either a distribution with support $\{\underline{U}\}$ (if $u_0 \leq \underline{U}$) or one with support $\{0, u_0\}$ (if $u_0 > \underline{U}$), with respective payoffs $(1-\mu)^{n-1}$ and

$(1-\mu) \times \underline{U}/u_0$. Therefore, full disclosure can arise in equilibrium if and only if

$$\frac{1-(1-\mu)^n}{n} \geq (1-\mu)^{n-1} \times \min\left\{1, \frac{\underline{U}}{u_0}\right\} . \tag{S4}$$

Recall that μ^{FD} is the unique solution to equation $1-(1-\mu)^n = n(1-\mu)^{n-1}$. Inequality (S4) holds whenever $\mu \geq \mu^{FD}$ regardless of u_0. When $\mu < \mu^{FD}$, inequality (S4) holds if and only if u_0 is sufficiently large; specifically:

$$u_0 \geq \frac{n(1-\mu)^{n-1}}{1-(1-\mu)^n} \times (\mu - c) \equiv u_0^{FD}(\mu) .$$

Note that u_0^{FD} is hump-shaped with $u_0^{FD}(c) = u_0^{FD}(1) = 0$. Therefore, for each $u_0 \in \left(0, \bar{U}\right)$, full disclosure can be sustained as an equilibrium either if μ is sufficiently large or if μ is sufficiently small.

We now move on to the partial disclosure equilibrium. Denote by v the equilibrium payoff of an individual firm, and by $\beta \in (0, 1-\mu)$ the atom at 0 that an individual firm assigns in its effective-value distribution. The two variables are related by

$$v = \frac{1-\beta^n}{n} . \tag{S5}$$

Suppose the equilibrium takes the semi-linear form. In this case, reservation value $\bar{U}$ is on the support and must deliver the equilibrium payoff v. Moreover, the atom β at 0 is due only to reservation value $\bar{U}$, and hence is equal to $\alpha(1-\mu)$. These two facts imply

$$v = (1-\mu)\frac{1-\left(1-\frac{\mu}{1-\mu}\beta\right)^n}{n\beta} . \tag{S6}$$

Equating (S5) and (S6) gives an equation in β, which has a unique solution in the interval $(0, 1-\mu)$ if and only if $\mu > 1/n$. To see this, note that the RHS of (S5) is decreasing and concave in β and equal to $1/n$ at $\beta = 0$, whereas the RHS of (S6) is decreasing and convex in β and equal to μ at $\beta = 0$. Moreover, it is straightforward to verify that the RHS of the two equations coincide when $\beta = 1-\mu$.

Suppose $\mu > 1/n$ and denote the unique solution (in the interval $\beta \in (0, 1-\mu)$) to the system of equations (S5) and (S6) above by $\left(\hat{v}, \hat{\beta}\right)$. Suppose further that $u_0 \leq \underline{U}$. As $\Pi_{\underline{U}}(w) = \hat{\Pi}_{\underline{U}}(w)$ for all $w \in [u_0, \underline{U}]$ (otherwise, this interval of effective values would not be on the support of the equilibrium distribution), it is necessary that

$$\frac{\hat{\beta}^{n-1}}{u_0} \geq \frac{\hat{v}}{\underline{U}} \Longleftrightarrow u_0 \leq \frac{\hat{\beta}^{n-1}}{\hat{v}}\underline{U} \equiv u_0^L(\mu) .$$

It is noteworthy that $u_0^L(\mu)$ equals 0 at $\mu = 1/n$, equals $\mu^{FD} - c$ at $\mu = \mu^{FD}$, and is increasing in μ. Moreover, it can be shown that $\hat{v} \geq \hat{\beta}^{n-1}$, so that $u_0^L(\mu) \leq \underline{U}$. In summation, the equilibrium can take the semi-linear form only if $\mu > 1/n$ and $u_0 \leq u_0^L(\mu)$. In this case, $\alpha = \hat{\alpha} \equiv \hat{\beta}/(1-\mu)$ and

$$\hat{U} - u_0 = \frac{\left(1 - \frac{\mu\hat{\beta}}{1-\mu}\right)^{n-1} - \hat{\beta}^{n-1}}{\hat{v} - \hat{\beta}^{n-1}} \left(\underline{U} - u_0\right) . \tag{S7}$$

The requirement $u_0 \leq u_0^L(\mu)$ ensures that $\frac{(\alpha(1-\mu))^{n-1}}{u_0} > \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}$ holds. We wish to establish that $\hat{U} \leq \bar{U}$. To this end, it suffices to focus on the case $u_0 = 0$, as $\hat{U}$ stated above is decreasing in u_0. As $\hat{v}$ and $\hat{\beta}$ are obtained by solving the system (S5) and (S6), the inequality $\hat{U} \leq \bar{U}$ can be stated as

$$\frac{1 - (1-\mu\hat{\alpha})^n}{n} \geq \mu\hat{\alpha}(1-\mu\hat{\alpha})^{n-1} + (\hat{\alpha}(1-\mu))^n , \tag{S8}$$

where $\hat{\alpha}$ is implicitly given by $\hat{\alpha}(\hat{\alpha}(1-\mu))^n + (1-\hat{\alpha}) = (1-\mu\hat{\alpha})^n$. It follows from a change of variable and straightforward algebra that (S8) can be rewritten as

$$T(x) \equiv \frac{\left(\frac{(1-x)x^{n-1}+1}{(1-x^n)} - \frac{1}{n}\right)^{-1} \left(\left(\frac{(1-x)x^{n-1}+1}{(1-x^n)} - \frac{1}{n}\right)^{-1} - 1 + x\right)^n}{x^n + \left(\frac{(1-x)x^{n-1}+1}{(1-x^n)} - \frac{1}{n}\right)^{-1} - 1} \leq 1 , \tag{S9}$$

where $x = 1 - \mu\hat{\alpha}$.[3] We show that (S9) holds for all $x \in \left[0, \mu^{FD}\right]$ and $n \geq 3$. It follows from direct substitution that $T(0) = 0$ and $T\left(\mu^{FD}\right) = 1$. It remains to show that $T(x)$ is increasing. By direct computation, $T'(x)$ has the same sign as $x^3 A(x) + x^{n+3} B(x)$, where

$$A(x) = (1-x^n)(n-1) - x^{n-2}\left(n^2 - n + 1\right)(1-x) + x^{n-1}(1-x) ,$$

and

$$B(x) = 1 - x^{n-3}(2x-1)(n + x - nx) .$$

We show that both $A(x)$ and $B(x)$ are nonnegative over $x \in \left[0, \mu^{FD}\right]$. First,

$$A'(x) = n^2 x^{n-3}(1-x)\left(x - \frac{n^3 - 3n^2 + 3n - 2}{n^2}\right) .$$

As $\left(n^3 - 3n^2 + 3n - 2\right)/n^2$ exceeds μ^{FD} for all $n \geq 3$, $A(x)$ is decreasing. Moreover, $A(0) = n - 1 > 0$ and $A(1) = 0$. Second,

$$B'(x) = -2(n-1)^2 x^{n-4}(1-x)\left(x - \frac{n(n-3)}{2(n-1)^2}\right) .$$

Therefore, B is either increasing or inverted U-shaped. Moreover, $B(0) = 1$ and $B(1) = 0$.

[3] Using the definition of $\hat{\alpha}$, the inequality $\hat{U} \leq \bar{U}$ can be written as $\hat{\alpha} \leq \left(\frac{(1-x)x^{n-1}+1}{(1-x^n)} - \frac{1}{n}\right)^{-1}$. Moreover, the implicit definition of $\hat{\alpha}$ can transformed into $x^n + \hat{\alpha} - 1 - \hat{\alpha}(\hat{\alpha} + x - 1)^n = 0$, yielding an inverse relation between $\hat{\alpha}$ and x. Moreover, as the LHS of the last equation is increasing in $\hat{\alpha}$, the inequality stated holds by substituting $\hat{\alpha} = \left(\frac{(1-x)x^{n-1}+1}{(1-x^n)} - \frac{1}{n}\right)^{-1}$.

Finally, consider equilibria that take the fully-linear form. With a full-linear payoff function, the equilibrium payoff v is equal to $\hat{\Pi}(\underline{U}; H_i)$, implying that

$$v = \frac{\beta^{n-1}}{u_0}\underline{U}\,. \tag{S10}$$

If the full information equilibrium exists, i.e., (S4) holds, the solution to the system of equations (S5) and (S10) would have $\beta > 1-\mu$ and $v < (1-(1-\mu)^n)/n$, eliminating this class of equilibria. Therefore, for the rest of this proof, suppose (S4) does not hold. It is clear that the system (S5) and (S10) has a unique solution–denote it by $\left(\hat{v}, \hat{\beta}\right)$. Define the probability of full information, $\hat{\alpha}$, as follows. If $\hat{v} \geq \mu$, set $\hat{\alpha} = 0$; otherwise, set $\hat{\alpha}$ to be the unique solution to $\hat{v} = \frac{1-(1-\alpha\mu)^n}{n\alpha}$.[4] Moreover, full-linearity dictates that $\hat{U} = (1-\hat{\alpha}\mu)^{n-1} \times \underline{U}/\hat{v}$. Evidently,

$$\hat{U} = (1-\alpha\mu)^{n-1} \times \frac{\underline{U}}{\hat{v}} = \frac{(1-\alpha\mu)^{n-1}}{\frac{1-(1-\alpha\mu)^n}{n\alpha}} \times \underline{U} = \frac{n\alpha\,(1-\alpha\mu)^{n-1}}{(1-(1-\alpha\mu)^n)} \times \underline{U} \leq \frac{1}{\mu} \times \underline{U} = \bar{U}\,,$$

where the inequality follows from the fact that $n\alpha\,(1-\alpha\mu)^{n-1}/(1-(1-\alpha\mu)^n)$ is a decreasing function in n and is equal to $1/n$ at $\alpha = 0$.

We finish by verifying that a fully-linear payoff function with $\alpha = \hat{\alpha}$ and $\hat{U}$ chosen above satisfies the necessary conditions for an equilibrium whenever $\mu \leq 1/n$ or $u_0 \geq u_0^L(\mu)$. To this end, it suffices to check that $\hat{\beta} \geq \hat{\alpha} \times (1-\mu)$. The case of $\hat{\alpha} = 0$ ($\hat{v} \geq \mu$) is immediate, so consider $\hat{\alpha} > 0$. As the RHS of (S5) is decreasing and concave in β, whereas the RHS of (S6) is decreasing and convex in β, either $\mu \leq 1/n$ or $u_0 \geq u_0^L(\mu)$ ensures that

$$\hat{v} \geq (1-\mu)\,\frac{1-\left(1-\frac{\mu}{1-\mu}\hat{\beta}\right)^n}{n\hat{\beta}}\,. \tag{S11}$$

The fact that $\frac{1-(1-\alpha\mu)^n}{n\alpha}$ is decreasing in α, together with the definition $\hat{v} = \frac{1-(1-\hat{\alpha}\mu)^n}{n\hat{\alpha}}$, implies $\hat{\beta}/(1-\mu) \geq \hat{\alpha}$.

The analysis above covers all parameter configurations for any $n \geq 3$, $\mu \in (0,1)$ and $u_0 \in \left(0, \bar{U}\right)$. ■

Similar to the no outside option case, the effective-value distributions that yield (S2) or (S3) can be generated by mixed strategies that involve randomization of binary distributions over posteriors only.

Lemma D.3. *The effective-value distribution H_i implied by either (S2) or (S3) is inducible. Moreover, it can be generated by a mixed strategy $(G(\cdot), \{F(\cdot; U) : U \in \operatorname{supp}(G)\})$ in which $F(\cdot; U)$ is binary for each $U \in \operatorname{supp}(G)$.*

Proof of Lemma D.3. Consider first the case $\mu \leq 1/n$ and $u_0 \in [u_0^L(\mu), u_0^{FD}(\mu))$, so that the equilibrium payoff function is fully linear. The effective-value distribution H_i implied by (S3) has an atom α at $\bar{U}$, an

[4] Note that failure of (S4) ensures that $\hat{\alpha} < 1$.

atom $(1-\alpha\mu)\left(\frac{u_0}{\hat{U}}\right)^{\frac{1}{n-1}}$ at 0, and a density

$$h_i(w)=\begin{cases}0 & \text{if} \quad w<u_0 \text{ and } w=(\hat{U},\bar{U}] \\ \frac{1-\alpha\mu}{(n-1)\hat{U}^{\frac{1}{n-1}}}w^{-\frac{n-2}{n-1}} & \text{if} \quad w\in\left[u_0,\hat{U}\right]\end{cases}.$$

Below, we construct a mixed strategy that generates this effective value distribution. To this end, define a mapping $\gamma:\ \left[\underline{U},\hat{U}\right]\longrightarrow[0,\underline{U}]$ by

$$K(\gamma(U))=K(U) \quad \text{for} \quad U\leq\tilde{U}, \quad \text{and} \quad \gamma(U)=0 \quad \text{for} \quad U\geq\tilde{U}\,,$$

where $K:\ \left[0,\bar{U}\right]\longrightarrow\mathbb{R}$ is defined as $K(w)\equiv(n\underline{U}-w)\,w^{\frac{1}{n-1}}$, $\tilde{U}>u_0$ is defined implicitly by $K\left(\tilde{U}\right)=K(u_0)$, and parameters α and $\hat{U}$ are as given in Lemma D.2. For each $U\in\left[\underline{U},\hat{U}\right]$, let $F(\cdot;U)$ be the binary distribution with support $\{\gamma(U),U\}$ and mean $\underline{U}$ and let $F\left(\cdot;\bar{U}\right)$ be the binary distribution with support $\left\{0,\bar{U}\right\}$ and mean $\underline{U}$. Moreover, let G be a reservation-value distribution that has an atom $\alpha\in[0,1]$ at $\bar{U}$ and a density as follows:

$$g(U)\equiv\frac{1-\alpha\mu}{(n-1)\,\hat{U}^{\frac{1}{n-1}}}\frac{U-\gamma(U)}{\underline{U}-\gamma(U)}U^{-\frac{n-2}{n-1}}, \quad \text{for} \quad U\in\left[\underline{U},\hat{U}\right].$$

Below, we show that the mixed strategy

$$\left(G,\left\{F(\cdot;U)\ :\ U\in\left[\underline{U},\hat{U}\right]\cup\left\{\bar{U}\right\}\right\}\right),$$

generates the effective-value distribution H_i defined above.

We need to establish that the effective-value distribution $F(\cdot;U)$ is inducible for each $U\in\left[\underline{U},\hat{U}\right]\cup\left\{\bar{U}\right\}$. First, the mapping γ is well-defined: a direct computation reveals that $K(w)$ is strictly concave with its peak at $\underline{U}$. Moreover, we can show that $\gamma(U)\leq a(U)\equiv\frac{\mu-c-\mu U}{1-c-U}$. For this purpose, it is without loss to suppose $u_0=0$, as $\gamma(U)$ defined above is weakly decreasing in u_0. Because $a(\underline{U})=\gamma(\underline{U})$, $\gamma\left(\hat{U}\right)=0=a\left(\bar{U}\right)\leq a\left(\hat{U}\right)$, and $a(U)$ is decreasing and strictly concave, it suffices to show that $\gamma(U)$ is convex. To this end, we adopt a change of variable: let $v=U-\underline{U}$, and $d(v)=\underline{U}-\gamma(\underline{U}+v)$. The implicit definition of γ implies $K(\underline{U}+v)=K(\underline{U}-d(v))$, or equivalently,

$$(\underline{U}+v)^{\frac{1}{n-1}}((n-1)\,\underline{U}-v)=(\underline{U}-d(v))^{\frac{1}{n-1}}((n-1)\,\underline{U}+d(v))\,.$$

The rest of the argument coincides with that in Lemma 4.2 in the text (after replacing M with $\underline{U}$).

We now check that the mixed strategy generates an effective-value distribution coinciding with H_i stated above. For $w\in\left[\underline{U},\tilde{U}\right]$, the density implied by the mixed strategy is

$$g(w)\times\frac{\underline{U}-\gamma(w)}{w-\gamma(w)}=\frac{1-\alpha\mu}{(n-1)\,\hat{U}^{\frac{1}{n-1}}}\frac{w-\gamma(w)}{\underline{U}-\gamma(w)}w^{-\frac{n-2}{n-1}}\times\frac{\underline{U}-\gamma(w)}{w-\gamma(w)}=h_i(w)\,.$$

For $w \in \left[\tilde{U}, \hat{U}\right]$, the density implied by the mixed strategy is

$$g(w) \times \frac{\underline{U}}{w} = \frac{1-\alpha\mu}{(n-1)\hat{U}^{\frac{1}{n-1}}} \frac{w}{\underline{U}} w^{-\frac{n-2}{n-1}} \times \frac{\underline{U}}{w} = h_i(w) \ .$$

Define $q: \ [0, \underline{U}] \longrightarrow \left[\underline{U}, \hat{U}\right]$ as the inverse mapping of γ. For $w \in [u_0, \underline{U}]$, the density implied by the mixed strategy is

$$\begin{aligned}
-q'(w) \times \frac{q(w)-\underline{U}}{q(w)-w} \times g(q(w)) &= -\frac{K'(w)}{K'(q(w))} \times \frac{q(w)-\underline{U}}{q(w)-w} \times g(q(w)) \\
&= -\frac{\frac{n}{n-1}(\underline{U}-w)\, w^{\frac{1}{n-1}-1}}{\frac{n}{n-1}(\underline{U}-q(w))\, q(w)^{\frac{1}{n-1}-1}} \times \frac{q(w)-\underline{U}}{q(w)-w} \times \frac{1-\alpha\mu}{(n-1)\hat{U}^{\frac{1}{n-1}}} \frac{q(w)-w}{\underline{U}-w} q(w)^{-\frac{n-2}{n-1}} \\
&= \frac{w^{\frac{1}{n-1}-1}}{q(w)^{\frac{1}{n-1}-1}} \times \frac{\underline{U}-w}{q(w)-w} \times \frac{1-\alpha\mu}{(n-1)\hat{U}^{\frac{1}{n-1}}} \frac{q(w)-w}{\underline{U}-w} q(w)^{-\frac{n-2}{n-1}} \\
&= \frac{1-\alpha\mu}{(n-1)\hat{U}^{\frac{1}{n-1}}} \times w^{-\frac{n-2}{n-1}} = h_i(w)
\end{aligned} .$$

The atom at 0 is given by

$$\begin{aligned}
\alpha(1-\mu) + \int_{\tilde{U}}^{\hat{U}} \left(1 - \frac{\underline{U}}{U}\right) dG(U) &= \alpha(1-\mu) + \int_{\tilde{U}}^{\hat{U}} \left(1 - \frac{\underline{U}}{w}\right) \times \left(\frac{1-\alpha\mu}{(n-1)\hat{U}^{\frac{1}{n-1}}} \frac{w}{\underline{U}} w^{-\frac{n-2}{n-1}}\right) dw \\
&= \alpha(1-\mu) + \frac{1-\alpha\mu}{n\underline{U}\hat{U}^{\frac{1}{n-1}}} \left(-\left(n\underline{U} - \hat{U}\right)\hat{U}^{\frac{1}{n-1}} + \left(n\underline{U} - \tilde{U}\right)\tilde{U}^{\frac{1}{n-1}}\right) \\
&= \alpha(1-\mu) + \frac{1-\alpha\mu}{n\underline{U}\hat{U}^{\frac{1}{n-1}}} \left(-\left(n\underline{U} - \hat{U}\right)\hat{U}^{\frac{1}{n-1}} + (n\underline{U} - u_0)\, u_0^{\frac{1}{n-1}}\right) \\
&= \alpha(1-\mu) + (1-\alpha\mu)\left(-\left(1 - \frac{(1-\alpha\mu)^{n-1}}{nv}\right) + \left(1 - \frac{\beta^{n-1}}{nv}\right)\frac{\beta}{1-\alpha\mu}\right) \\
&= \beta + \frac{(1-\alpha\mu)^n - \beta^n - (1-\alpha)\, nv}{nv} = \beta
\end{aligned} .$$

where the first equality uses the definition of g, the third equality uses the definition of $\tilde{U}$, the fourth equality uses the linearity of the payoff function: $v/\underline{U} = \beta^{n-1}/u_0 = (1-\alpha\mu)^{n-1}/\hat{U}$, and the last equality uses the fact that $v = \frac{1-\beta^n}{n}$ and $v = \frac{1-(1-\alpha\mu)^n}{n\alpha}$ (in the case $\alpha > 0$).

Consider next the case $\mu > 1/n$ and $u_0 \in [0, u_0^L(\mu))$, so that the equilibrium payoff function is semi-linear. The effective-value distribution H_i implied by (S2) has an atom α at $\bar{U}$, an atom $\alpha(1-\mu)$ at 0, and a density

$$h_i(w) = \begin{cases} 0 & \text{if} \quad w < u_0 \text{ and } w = (\hat{U}, \bar{U}] \\ \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{(n-1)\left(\hat{U} - u_0\right)} \left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(w - u_0)\right)^{-\frac{n-2}{n-1}} & \text{if} \quad w \in \left[u_0, \hat{U}\right] \end{cases} .$$

Define a mapping $\gamma : \left[\underline{U}, \hat{U}\right] \longrightarrow [0, \underline{U}]$ by $K(\gamma(U)) = K(U)$, where $K : \left[0, \bar{U}\right] \longrightarrow \mathbb{R}$ is given by

$$K(w) \equiv \left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(w - u_0)\right)^{\frac{1}{n-1}} \times \left(n\underline{U} - (n-1)u_0 + (n-1)\left(\frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}\right)^{-1}(\alpha(1-\mu))^{n-1} - w\right),$$

and parameters α and $\hat{U}$ are as given in Lemma D.2. For each $U \in \left[\underline{U}, \hat{U}\right]$, let $F(\cdot; U)$ be a binary distribution with support $\{\gamma(U), U\}$ and mean $\underline{U}$, and let $F\left(\cdot; \bar{U}\right)$ be the binary distribution with support $\left\{0, \bar{U}\right\}$ and mean $\underline{U}$. Moreover, let G be a reservation-value distribution that has an atom $\alpha \in [0, 1]$ at $\bar{U}$ and density

$$g(U) \equiv \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{(n-1)\left(\hat{U} - u_0\right)}\left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(U - u_0)\right)^{-\frac{n-2}{n-1}} \times \frac{U - \gamma(U)}{\underline{U} - \gamma(U)}, \quad \text{for } U \in \left[\underline{U}, \hat{U}\right].$$

We need to establish that effective-value distribution $F(\cdot; U)$ is inducible for each $U \in \left[\underline{U}, \hat{U}\right] \cup \left\{\bar{U}\right\}$. First, the mapping γ is well-defined: a direct computation reveals that $K(w)$ is strictly concave with its peak at $\underline{U}$ and that $K\left(\hat{U}\right) = K(u_0)$. Moreover, we can show that $\gamma(U) \le a(U) \equiv \frac{\mu - c - \mu U}{1 - c - U}$. To this end, note that because $a(\underline{U}) = \gamma(\underline{U})$, $\gamma\left(\hat{U}\right) = u_0 \le a\left(\hat{U}\right)$,[5] and $a(U)$ is decreasing and strictly concave, it suffices to show that $\gamma(U)$ is convex. To this end, we adopt a change of variable: let $v = U - \underline{U}$, and $d(v) = \underline{U} - \gamma(\underline{U} + v)$. The implicit definition of γ implies $K(\underline{U} + v) = K(\underline{U} - d(v))$, or equivalently,

$$(L + v)^{\frac{1}{n-1}}((n-1)L - v) = (L - d(v))^{\frac{1}{n-1}}((n-1)L + d(v)),$$

where $L \equiv (\underline{U} - u_0) + \frac{\left(\hat{U} - u_0\right)(\alpha(1-\mu))^{n-1}}{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}$. The rest of the argument coincides with that for Lemma 4.2 in the text (after replacing M with L).

Let us now check that the mixed strategy generates an effective-value distribution coinciding with H_i stated above. For $w \in \left[\underline{U}, \hat{U}\right]$, the density implied by the mixed strategy is

$$\begin{aligned} g(w) \times \frac{\underline{U} - \gamma(w)}{w - \gamma(w)} &= \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{(n-1)\left(\hat{U} - u_0\right)}\left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(w - u_0)\right)^{-\frac{n-2}{n-1}} \\ &\times \left(\frac{w - \gamma(w)}{\underline{U} - \gamma(w)}\right)\left(\frac{\underline{U} - \gamma(w)}{w - \gamma(w)}\right) = h_i(w) \end{aligned}$$

[5]The reason is as follows. The derivation in Lemma D.2 reveals that $\hat{U} \le \bar{U}$ in this parameter range, including the case $u_0 = 0$, so $\gamma^{-1}(0) \le \bar{U}$. Moreover, we have established in the case of the fully-linear equilibrium above that $\gamma\left(\hat{U}\right) \le a\left(\hat{U}\right)$ holds at $u_0 = u_0^L(\mu)$, so $\gamma^{-1}\left(u_0^L(\mu)\right) \le a^{-1}\left(u_0^L(\mu)\right)$. Furthermore, for all $u_0 \in [0, u_0^L(\mu))$, $\hat{U}$ varies linearly with u_0. Together with the fact that a is strictly concave, we have $\gamma^{-1}(u_0) < a^{-1}(u_0)$ for all $u_0 \in [0, u_0^L(\mu))$.

Let q be the inverse mapping of γ. For $w \in [u_0, \underline{U}]$, the density implied by the mixed strategy is

$$
\begin{aligned}
& -q'(w) \times \frac{q(w) - \underline{U}}{q(w) - w} \times g(q(w)) \\
= & -\frac{K'(w)}{K'(q(w))} \times \frac{q(w) - \underline{U}}{q(w) - w} \times g(q(w)) \\
= & -\frac{(\underline{U} - w)\left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(w - u_0)\right)^{-\frac{n-2}{n-1}}}{(\underline{U} - q(w))\left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(q(w) - u_0)\right)^{-\frac{n-2}{n-1}}} \times \frac{q(w) - \underline{U}}{q(w) - w} \times g(q(w)) \\
= & \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{(n-1)\left(\hat{U} - u_0\right)}\left((\alpha(1-\mu))^{n-1} + \frac{(1-\alpha\mu)^{n-1} - (\alpha(1-\mu))^{n-1}}{\hat{U} - u_0}(w - u_0)\right)^{-\frac{n-2}{n-1}} = h_i(w)
\end{aligned}
.
$$

■

The introduction of a relevant outside option allows us to investigate factors that can affect industry profits. Interestingly, industry profits can be hurt not only by an increase in the consumer's search cost c, but also by an improvement in the average product quality μ. These comparative statics illustrate nicely the signal's dual role of attraction and persuasion. On the one hand, an improvement in μ facilitates persuasion, as it lifts the posterior quality realization on average. In fact, it is easy to see that absent any competition, a firm would unambiguously benefit from having a higher μ, as it would allow the firm to increase the probability of realizing a posterior quality above u_0. On the other hand, with a higher μ, the signal's role as an instrument of attraction becomes ever more important, as the chance that the consumer visits low-ranking firms dwindles. A higher average quality thus incites more aggressive information revelation–which harms profits–by lowering the probability that the consumer makes a purchase. We find that the former effect is more important when μ is relatively low, but the latter effect dominates when μ is relatively high.

A standard prediction from the literature on random consumer search (e.g., Wolinsky (1986) and Anderson and Renault (1999)) is that a higher search cost increases profits, as it raises the likelihood that consumers stop and purchase conditional on visiting a firm, thus softening the market competition. In our setting, the firms' signal choices direct the consumer's search, and an increase in the search cost is bad news for the firms, as the consumer is less willing to visit them in the first place. In equilibrium, firms respond by disclosing more aggressively, which results in a higher likelihood that the consumer takes up her outside option, and thus lower industry profits.

Corollary D.4. *(i) Suppose $u_0 < \mu^{FD} - c$. There exists a $\mu^* < \mu^{FD}$ such that a firm's equilibrium profit is increasing in μ for all $\mu < \mu^*$, and decreasing in μ for $\mu \in (\mu^*, \mu^{FD})$.*

(ii) A firm's equilibrium profit is weakly decreasing in the consumer's search cost c, strictly so if u_0 is in some intermediate region.

Proof. (i) Fix a $u_0 < \mu^{FD} - c$ and let μ^* be the unique solution to $u_0^L(\mu) = u_0$. We will show that a firm's equilibrium profit is increasing in μ when the equilibrium takes the fully linear form (i.e., $\mu < \mu^*$) and is decreasing in μ when the equilibrium takes the semi-linear form.

Consider first the case where the equilibrium payoff is fully linear. Recall from the proof of Lemma D.2 that a firm's equilibrium profit v is jointly determined by ($S5$) and ($S10$). As the RHS of ($S5$) is decreasing in β and the RHS of ($S10$) is increasing in β, and because an increase in μ shifts up the RHS of ($S10$), the implied equilibrium payoff v is therefore increasing in μ.

Consider next the case where the equilibrium payoff is semi-linear. Recall from the proof of Lemma D.2 that a firm's equilibrium profit v and the atom β at the bottom is jointly determined by equating ($S5$) and ($S6$), i.e., $T(\beta, \mu) = 0$, where

$$T(\beta, \mu) \equiv (1-\mu) \frac{1 - \left(1 - \frac{\mu}{1-\mu}\beta\right)^n}{n\beta} - \frac{1-\beta^n}{n}.$$

It is straightforward to verify that T is strictly convex in β, is positive at $\beta = 0$ and equals 0 at $\beta = 1 - \mu$. The root that is smaller than $1-\mu$ thus gives the equilibrium value of β. It is immediate that T is increasing in μ, and so is the equilibrium value of β.[6] As the equilibrium payoff is decreasing in β (recall ($S5$)), it is also decreasing in μ.

(ii) The proof of Lemma D.2 implies the search cost c has no impact on a firm's profit v if the equilibrium payoff function is semi-linear, or if the equilibrium involves full disclosure. In the case of a fully-linear equilibrium payoff, v is jointly determined by ($S5$) and ($S10$). Evidently, ($S5$) is independent of c whereas the RHS of ($S10$) is decreasing in c. An increase in c thus lowers the equilibrium value of v. ■

E Attraction Versus Persuasion for a Prior with a Density

In this section, we establish that the qualitative findings from Au and Whitmeyer (2021) extend to a setting in which the consumer's valuation is continuously distributed on some interval. As an illustration, in

[6] Direct computation shows that T is increasing in μ if and only if $(1-\mu)\left(1 - \left(1 - \frac{\mu}{1-\mu}\beta\right)^n\right)/n\beta < \left(1 - \frac{\mu}{1-\mu}\beta\right)^{n-1}$ holds for all $\beta \in (0, 1-\mu)$. Letting $\alpha = \beta/(1-\mu)$, the last inequality can be equivalently expressed as $(1-\alpha\mu)^n + n\alpha(1-\alpha\mu)^{n-1} - 1 > 0$ for all $\alpha \in (0,1)$. The last inequality holds because its LHS is inverted U-shaped in α, equal to 0 at $\alpha = 0$, and is positive at $\alpha = 1$ (as long as $\mu < \mu^{FD}$).

subsection E.1, we explore in detail the case in which the consumer's prior for a product is uniformly distributed on the unit interval.

Formally, let the consumer's valuation for each firm i be a random variable X_i drawn independently and identically from some cumulative distribution function $\Phi = \Phi_i$ that is supported on $[0, 1]$. We assume that Φ has a continuously differentiable density φ, and that φ' is bounded (which, therefore, implies that φ is bounded). The consumer has an outside option of 0, and so we also impose that $\mathbb{E}_{\Phi}[X_i] \geq c$, since otherwise a market would not exist (the consumer would take her outside option with probability 1, regardless of the firms' information provision policies).

Because the consumer is risk-neutral, each firm's problem of choosing a signal is equivalent to one in which it chooses a distribution over posteriors $F \in \mathcal{F}(\Phi)$, where $\mathcal{F}(\Phi)$ is the set of mean-preserving contractions of Φ. Our main result establishes that (i) if a pure strategy equilibrium exists, it must be one in which firms induce the maximal reservation value; and (ii) such an equilibrium exists only if competition is sufficiently fierce.

The maximal reservation value, $\bar{U}_{\Phi}$, is defined implicitly as

$$c = \int_{\bar{U}_{\Phi}}^{1} \left(x - \bar{U}_{\Phi}\right) d\Phi(x) \ .$$

Note that, in contrast to the binary prior setting, the maximal reservation value is not induced uniquely by the prior Φ. Any mean-preserving contraction of Φ, F, that is equal to Φ on $\left[\bar{U}_{\Phi}, 1\right]$ induces $\bar{U}_{\Phi}$. Then,

Proposition E.1. *(i) If there does not exist a symmetric pure-strategy equilibrium in which firms induce the maximal reservation value, $\bar{U}_{\Phi}$, there exist no symmetric pure-strategy equilibria.*

(ii) There exists a positive $N \in \mathbb{N}$ such that if $n \geq N$ there exists a unique symmetric pure strategy equilibrium in which each firm provides full information.

(iii) There exists a positive $M \in \mathbb{N}$ ($M \leq N$) such that if $n \geq M$ there exists a unique symmetric pure strategy equilibrium in which each firm induces $\bar{U}_{\Phi}$.

Proof. The first statement is analogous to the corresponding result for a binary prior: in any purported equilibrium in which each firm induces some reservation value $U < \bar{U}_{\Phi}$ a firm can always deviate profitably by providing slightly more information and moving to the top of the consumer's search order.

Now let us prove the second statement. Because φ' is bounded, there exists a positive $\tilde{N} \in \mathbb{N}$ such that for all $n \geq \tilde{N}$, Φ^{n-1} is convex on $\left[0, \bar{U}_{\Phi}\right]$. For all such n, no firm can deviate profitably by choosing any distribution that induces reservation value $\bar{U}_{\Phi}$ (this follows from Corollary 1 in Hwang, Kim, and Boleslavsky (2018)).

We now need to check whether a firm can deviate profitably by choosing a distribution that induces a reservation value that is strictly less than $\bar{U}_\Phi$. For any such deviation, the firm will be visited last, so its payoff as a function of its realized value, x, is

$$V(x) = \begin{cases} \Phi(x)^{n-1}, & \text{if} \quad 0 \leq x \leq \bar{U}_\Phi \\ \Phi\left(\bar{U}_\Phi\right)^{n-1}, & \text{if} \quad \bar{U}_\Phi \leq x \leq 1 \end{cases}.$$

If $\bar{U}_\Phi \leq \mu$, providing no information is optimal. This yields a payoff of $\Phi\left(\bar{U}_\Phi\right)^{n-1}$. Because $\Phi\left(\bar{U}_\Phi\right) < 1$ there exists a finite number $\hat{N}$ such that for all $n \geq \hat{N}$, $\Phi\left(\bar{U}_\Phi\right)^{n-1} \leq 1/n$.

If $\bar{U}_\Phi > \mu$ the deviator's payoff is obviously bounded above by $\Phi\left(\bar{U}_\Phi\right)^{n-1}$ and so again, for all $n \geq \hat{N}$ there is no profitable deviation. We may define $N \equiv \max\left\{\tilde{N}, \hat{N}\right\}$ and conclude the second part of the result.

If n is such that Φ^{n-1} is not convex on $\left[0, \bar{U}_\Phi\right]$ then firms do not provide full information in a symmetric pure-strategy equilibrium, which follows from Hwang, Kim, and Boleslavsky (2018). Instead (if such an equilibrium exists) each firm chooses an F that induces $\bar{U}_\Phi$ such that F is an alternating $(n-1)$-MPC[7] of Φ and F^{n-1} is convex on $[0, \bar{U}_\Phi]$. This follows from Theorem 1 in Hwang, Kim, and Boleslavsky (2018). Note that the portion of F below $\bar{U}_\Phi$ is unique. For the remainder of the proof, observe that by construction F^{n-1} is convex and so the previous analysis can be repeated. ■

This proposition highlights one special feature of the binary prior case; that the maximum reservation value is induced *uniquely* by the prior. More generally, the level of competition required for firms to provide full information in the (unique) symmetric equilibrium is weakly higher than that required for them to induce the maximum reservation value. If there is a gap between these thresholds, in between the firms fully reveal when the consumer's match quality is above the maximum reservation value (each firm's distribution F equals the prior, Φ, above $\bar{U}_\Phi$) but partially obfuscate match values below $\bar{U}_\Phi$.

E.1 Uniform Prior Example

Suppose the prior is the uniform distribution on the unit interval: $\Phi = \mathcal{U}[0,1]$ and that $c < 1/2$. We find that there exists an equilibrium in which firms provide full information for any number of firms $n \geq 2$.

Proposition E.2. *For any number of firms $n \geq 2$ there exists a unique pure strategy equilibrium. Each firm provides full information.*

[7] Refer to Hwang, Kim, and Boleslavsky (2018) for a definition of this term.

Proof. The maximal reservation is $\bar{U}_{\mathcal{U}} = 1 - \sqrt{2c}$, which is easy to compute. Henceforth, we omit this subscript. Because $\Phi^{n-1} = x^{n-1}$ is convex, no firm can deviate profitably by choosing another distribution that induces $\bar{U}$ (we dropped the subscript).

Accordingly, we need only check that a firm cannot deviate profitably by choosing a distribution that induces a reservation value that is strictly less than $\bar{U}$. Its payoff as a function of its realized value is

$$V(x) = \begin{cases} x^{n-1}, & \text{if} \quad 0 \leq x \leq 1 - \sqrt{2c} \\ \left(1 - \sqrt{2c}\right)^{n-1}, & \text{if} \quad 1 - \sqrt{2c} \leq x \leq 1 \end{cases}.$$

If $1 - \sqrt{2c} \leq 1/2$, providing no information is optimal, which yields the payoff

$$\left(1 - \sqrt{2c}\right)^{n-1} \leq \left(\frac{1}{2}\right)^{n-1} \leq \frac{1}{n},$$

for all $n \geq 2$. Thus, there are no profitable deviations for c sufficiently high. If $1 - \sqrt{2c} > 1/2$, things are a little trickier, but Dworczak and Martini (2019) provide the tools to establish the result. Indeed, the corresponding price function (in their parlance) is

$$P(x) \equiv \begin{cases} x^{n-1}, & 0 \leq x \leq 1 - 2\sqrt{2c} \\ \frac{\left(1-\sqrt{2c}\right)^{n-1} - \left(1-2^{\frac{3}{2}}\sqrt{c}\right)^{n-1}}{\sqrt{2c}} x + \left(1 - 2^{\frac{3}{2}}\sqrt{c}\right)^{(n-1)} - \frac{\left(1-\sqrt{2c}\right)^{n-1}\left(1-2^{\frac{3}{2}}\sqrt{c}\right) - \left(1-2^{\frac{3}{2}}\sqrt{c}\right)^{n}}{\sqrt{2c}}, & 1 - 2\sqrt{2c} \leq x \leq 1 \end{cases}.$$

This is depicted in Figure 1. Consequently, the deviator's maximal payoff is

$$\int_0^{1-2\sqrt{2c}} x^{n-1} dx + 2\sqrt{2c}\left(1 - \sqrt{2c}\right)^{n-1} = \frac{\left(1 - 2\sqrt{2c}\right)^n}{n} + 2\sqrt{2c}\left(1 - \sqrt{2c}\right)^{n-1} \leq \frac{1}{n},$$

which concludes the proof. ■

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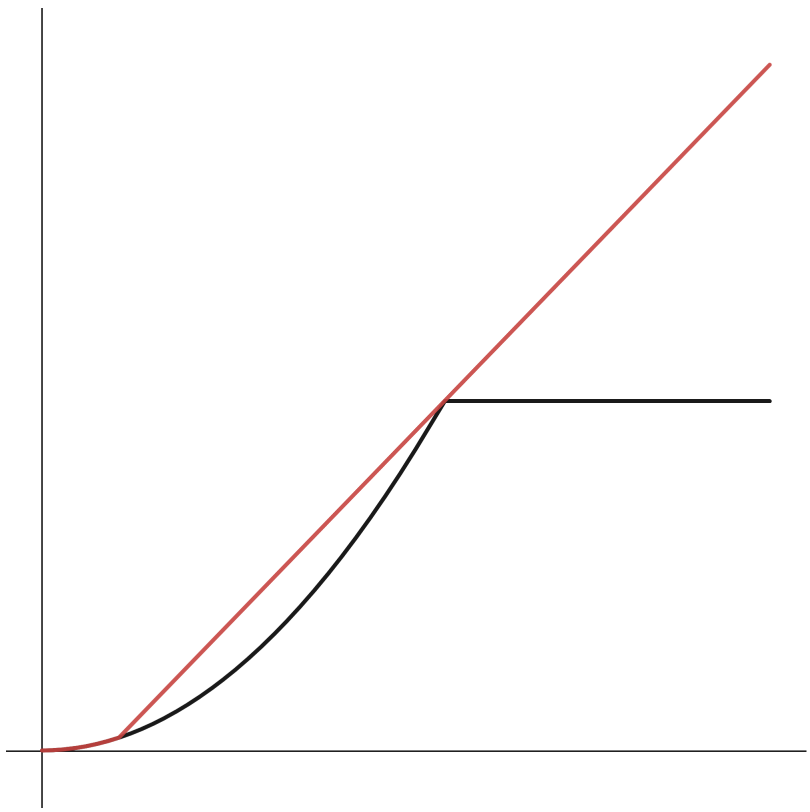

Figure 1: V (black) and P (red) for $F = \Phi = \mathcal{U}[0, 1]$, $c = .1$, and $n = 3$.

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