Search and Competition with Endogenous Investigations

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Abstract

We modify the standard model of price competition with horizontally differentiated products, imperfect information, and search frictions by allowing consumers to flexibly acquire information about a product's match value during their visits. We characterize a consumer's optimal search and information acquisition protocol and analyze the pricing game between firms. Search frictions and information frictions affect market prices, profits, and consumer welfare in markedly different ways. Although higher search costs beget higher prices (and profits for firms), higher information acquisition costs lead to *lower* prices and may benefit consumers. We discuss implications of our findings for policies concerning disclosure rules and hidden fees.

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1 Introduction

There is a large and rich literature on sequential consumer search.[1] Many papers in this area share a similar common structure: there are collections of possibly heterogeneous firms and consumers, the latter of which wish to purchase products from the firms. These consumers (or at least some subset of them) visit firms in sequence in order to discover important information about the products including, e.g., their qualities, if the products are vertically differentiated; match values, if there is horizontal differentiation; and/or prices. The essential decisions made by consumers are whom to visit (if search is directed), when to stop, and whether to purchase upon stopping (and from whom).

Despite the considerable control they possess regarding their search behavior, consumers in these models are surprisingly passive in a different sense. Namely, when a consumer visits a firm she is unable to affect the information her visit reveals–there is an equivalence between visiting a firm and acquiring information about its product(s). She observes (at least some of) the pertinent information (prices, qualities, and/or match values) or the draw of some signal correlated with these values, but cannot control this information *directly*.

In many settings this is unrealistic. Consider, for instance, a prospective homeowner looking for a house. She visits homes on the market in sequence; but rather than absorb information about a house effortlessly upon visiting it, she must acquire this information actively. Not only does the consumer face a sequential search problem as she moves from house to house, but within each visit, she has a dynamic information acquisition problem. She chooses which rooms to visit and how long to linger in each; what questions to ask the real-estate agent; how thoroughly to inspect the plumbing, lighting, or paint condition; and how many dark corners to scrutinize for mold.[2]

This phenomenon is not limited merely to the housing market. Indeed, it is difficult

[1] Seminal papers include Wolinsky (1986), Stahl (1989), and Anderson and Renault (1999). Anderson and Renault (2018) provide a nice overview of consumer search, both sequential and simultaneous.

[2] In fact there are a number of guides online that advise consumers how to behave optimally during such visits; one urges its readers, "Once inside a home, try everything. Follow common courtesy but don't be shy." (https://www.redfin.com/home-buying-guide/what-to-look-for)

to think of many markets in which such information acquisition does not take place. A prospective car buyer chooses how many features of a car to inspect when at a dealer, a consumer shopping for groceries chooses how much of each product's label to peruse, a teenager looking for a prom dress scrutinizes candidates in the many mirrors of a dress store's changing room, and a music enthusiast looking for new headphones plays various songs to test out different facets of a device's sound quality and feel.

Our goal in this paper is to explore the effect of this flexible information acquisition on consumer and firm behavior in an otherwise standard sequential search market. There are a large number of consumers with unit demand for a horizontally differentiated product and a large mass of *ex ante* indistinguishable firms that sell the product. It costs a consumer $c > 0$ to visit a firm; and when at a firm she acquires information about the product's idiosyncratic fit by sampling various "attributes"–which inform her valuation–in sequence, at a cost of $\gamma > 0$ per attribute.

The firms compete by setting prices, which can only be discovered by the consumer upon visiting a firm. In our main analysis we assume that when visiting a firm, a consumer observes the firm's price before acquiring information (before she starts sampling attributes at the firm). In many circumstances this is realistic: most retailers of consumer goods post prices that are non-negotiable and which consumers note before inspecting the items. Due to the symmetry of the model we focus on symmetric equilibria.

Within this setting, we tackle a number of questions. First, it is a standard result that in search models with hidden prices, an increase in the search cost leads to an increase in prices. Does this hold when there are information frictions as well? What about the effect of information frictions on prices? That is, do greater information frictions lead to higher prices? Do laws mandating maximal disclosure always benefit consumers? Do they always hurt firms?

Crucially, a consumer's optimal information acquisition protocol is shaped by the prices set by firms. This gives her considerable power to react to a price deviation by a firm. Even though she can not observe a price change before her visit, such a deviation will affect her optimal information acquisition protocol at that firm. This is realistic–consumers, upon seeing an unexpectedly high price, may just decide that the product is too expensive to consider at all.

We find that (explicit) search costs continue to have the same effect as in models without flexible learning. That is, prices and firm profits increase in (explicit) search costs, whereas consumer welfare decreases. On the other hand, as in the literature that explores a monopolist's pricing problem to a consumer who learns flexibly, the effect of information frictions is ambiguous. Prices decrease in the level of information frictions, and so consumer welfare may rise as a result. In fact, for certain regions of the parameter universe, an increase in information frictions leads to a Pareto increase in welfare. The expected duration of search is also non-monotone in the amount of information frictions.

One notable implication of our results is that a firm's market power is generated entirely by the search costs (think switching costs) and not by the information (learning) costs. To elaborate, it is only the existence of this search cost that prevents a consumer from departing the firm at the first sign of bad news. The information frictions, in turn, generate a downward sloping demand curve for firms, which becomes steeper (*ceteris paribus*) as information frictions increase. Thus, an increase in information frictions results in a decrease in the equilibrium price, and this decrease may be so great as to outweigh a consumer's direct welfare loss as a result of increased frictions. Firms are unaffected by the change in information frictions, since their profits rely on the market power granted to them by the search cost.

Later on we explore a natural modification of the model: we stipulate that a consumer does not observe a firm's price until *after* she has acquired information. This leads to an "informational hold-up problem," and in the unique equilibrium, firms mix over prices. Thus, our model generates price dispersion even when prices are not posted. The qualitative results from the main setting continue to hold: explicit search costs benefit firms at the expense of consumers but information frictions may be Pareto improving.

We also compare the results from the two timing specifications. Consumers strictly prefer that prices be observable before learning. Firms, on the other hand, may prefer that prices be hidden, but only if the expected quality of the product and the level of information frictions are sufficiently high. Otherwise, firms prefer that prices be observable as well. The comparisons in this section allow us to speak to the recent debate about hidden fees, drip pricing, and other varieties of hidden prices. Our results suggest that laws that mandate transparent prices do indeed benefit consumers, since they allow them to learn efficiently. Moreover, firms may benefit from such laws as well.

We finish this section by discussing the relevant papers in the literature. Section 2 lays out the model and establishes some preliminary results. Section 3 characterizes a consumer's optimal search and information acquisition protocol, and Section 4 explores the equilibrium in the pricing game between the firms. In Section 5 we modify the game so that prices are now unobservable to the consumer before she acquires information and we look at the pricing game between the firms. Section 6 compares welfare in the two timing regimes and Section 7 concludes. All proofs are left to the appendix,[3] unless otherwise noted.

1.1 Related Work

The paper closest to this one is Branco et al. (2012),[4] who look at the monopolist's problem when consumers flexibly acquire information after observing the monopolist's price. As in our search setting, they find that information frictions may Pareto-improve welfare. The portion of our paper in which prices are not observable before consumers acquire information is related to Ravid et al. (2020), who explore a bilateral-trade setting in which a consumer acquires costly information about a monopolist's product before observing the seller's price. Their focus is on the limiting case as information frictions vanish, and they establish that in the limit, the equilibrium converges to the Pareto-worst free learning equilibrium.[5] Also relevant is Dogan and Hu (2018), who explore the buyer-optimal

[3]Due to the algebra-heavy nature of some of the comparative statics derivations, we also have a Supplementary Appendix that provides Mathematica code to corroborate our calculations.

[4]We discuss their model in detail in the next section, when we describe our setup. Henceforth, we refer to them as BSV.

[5]A consumer's learning problem in our setting is not quite the same as that in Ravid et al. (2020). In their model the consumer faces a static problem: she chooses a signal about the value of a product, about which she has some prior distribution. As shown by Morris and Strack (2017), when the prior is binary there is an equivalence between the static information acquisition problem, in which a decision-maker chooses a distribution over posteriors subject to a posterior-separable cost, and a dynamic Wald stopping

signal in the random sequential search setting of Wolinsky (1986).

Other papers that look at a monopolist selling to consumers who may acquire information flexibly include Branco et al. (2016), Pease (2018), and Lang (2019). In the latter two papers, a consumer observes a monopolist's posted price before observing a diffusion process correlated with the product's quality at a flow cost. Lang (2019) assumes that the consumer's prior valuation for the product is distributed normally and finds that in regions of the parameters with active information acquisition, increased information frictions improve the monopolist's profit if and only if the *ex ante* expected value for the product is sufficiently high. Pease (2018) assumes a binary value for the product, and also finds that the level of information frictions has an ambiguous effect on firm profit. Branco et al. (2016) modify the setting of BSV by allowing the seller to choose how much information to provide about the product and find that there may be overprovision of information.

To our knowledge, the only other work to explore the dual effects of information and search frictions in a sequential search model is Guo (2021). In his model, a consumer searches sequentially and when at a firm draws her value from a family of rotation ordered distributions. The primary focus of his paper is the case in which a consumer chooses from a collection of uniform distributions. There, he finds that the explicit search cost and the cost of informativeness (in the rotation order) of a consumer's posterior value distribution operate in similar ways: equilibrium price and profit increase in both types of friction but may drop discontinuously for high frictions (of either variety). The reverse happens for consumer welfare. One important difference between our setups is that our consumers tackle a sequential information acquisition problem.

The literature that explores more standard versions of the hold-up problem in bilateral trade is also relevant. Recently, Dilmé (2019) looks at a scenario in which a seller can privately invest in the quality of its product before a buyer makes a take-it-or-leave-it offer. As we find in the hidden prices portion of this paper, and following similar logic, Dilmé shows that the seller randomizes over investment levels. Rao (2021) embeds a

problem. Beyond the binary-prior setting, this equivalence vanishes. Moreover, our problem is not a Wald problem–a consumer directly observes various components that affect her expected value for the product.

similar hold-up problem into a labor search model, in which workers invest in human capital. With hidden investment, Rao shows that the equilibrium exhibits both wage and skill dispersion akin to our information and price dispersion.

Naturally, our paper is also related to the broader collection of works that look at the effects of consumer rational inattention on market behavior. Of particular pertinence is Matějka and McKay (2012), who explore a model in which a consumer must incur a cost to evaluate the offers of firms within a market. Notably, they show that prices are increasing (and, hence, consumer welfare decreasing) in the size of the information frictions. Also related is Liu and Dukes (2016), who look at a simultaneous search model, in which a consumer must evaluate products in an oligopolistic market subject to an evaluation cost. Crucially, the dynamic aspect of our problem–in which a consumer's optimal behavior (both regarding her search behavior and her information acquisition protocol) is determined each period–allows a consumer to react to price deviations by firms. They find that an increase in information costs may benefit consumers, though at the expense of total welfare. Ursu et al. (2020) investigates theoretically and empirically a search model that allows consumers to inspect the same product multiple times.

2 The Model

Our model extends the framework of Branco et al. (2016) (BSV) to a random search setting. There is a continuum (unit mass) of selling firms and a continuum (unit mass) of consumers. Each firm's product has T attributes that are *ex ante* unknown. The value of attribute j to a consumer is an i.i.d. random variable X_j that takes values $-z$ and z with equal probability, where $z > 0$. If firm i's price is p_i then a consumer's realized utility from purchasing product i is $\mu - p_i + \sum_{j=1}^{T} X_j$, where $\mu \in \mathbb{R}$ is the consumer's baseline value for the products for sale (which is the same for each firm). Given this, it is the idiosyncratic attributes of a firm's product that set it apart from its competitors.

In the first period, a consumer is randomly matched with a firm whose attributes she can sequentially learn at a cost of $\gamma > 0$ per attribute. After examining t attributes, her expected valuation from purchasing firm i's product is $v_i(t) - p_i = \mu - p_i + \sum_{j=1}^{t} x_j$. We continue to follow BSV in assuming that the change in utility from each attribute, z, becomes infinitesimal as T grows infinitely large. This captures the fact that the importance of any one attribute is negligible compared to the overall value of the product. In the limit, the $v_i(t)$ process becomes a Brownian motion–$dv = \sigma dW$, where W is a standardized Brownian motion–with standard deviation σ, where $z = \sigma\sqrt{dt}$.[6] At each instant, the consumer decides whether to keep observing attributes or stop doing so. Once she stops, she has three options: she can purchase from the firm, or select her outside option with value 0, or be randomly matched with another firm where she faces an identical problem. As is routine in the literature, the first period visit is free, but any subsequent search imposes a cost $c > 0$ per period. To obtain equilibria with active search, we assume throughout that $c < \sigma^2/(4\gamma)$. For tractability, we focus on equilibria where firms play symmetric pure strategies. As is standard, we require that if the consumer observes a deviation at a firm, her beliefs about the other firms' prices are unchanged. [7]

The timing of the game is summarized in Figure 1 below.

2.1 Simplifying the Consumer's Information Acquisition Problem

Next, we observe that the consumer's information acquisition problem can be reformulated from a dynamic optimal stopping problem to a much simpler static problem. Evidently, at a firm, a consumer's information acquisition strategy is a stopping time τ, and any such τ generates a distribution over (expected values) v, which we denote F_τ. That is,

$$F_\tau(v) := \mathbb{P}(V_\tau \leq v) \ .$$

[6]As noted by Lang (2019), there is a subtle technical issue to this approach in that the "actual value for the product $\lim_{t\to\infty}\int_0^t W_s ds$ is ill-defined." However, we argue that this modeling choice is akin to the use of the improper uniform distribution in the global games literature, and which greatly helps us with simplicity at the cost of slight impropriety. Furthermore, for most of this paper, we use a convenient reformulation of the consumer's stopping problem as a static information acquisition problem, and we urge any objectors to the dynamic set-up to view it exclusively as the static problem instead.

[7]This is a standard assumption in the literature, though one that may not be realistic in situations with vertical relations (see, e.g., Janssen and Shelegia (2020)). However, these are absent from our model, so we claim that our handling of off-path beliefs is appropriate.

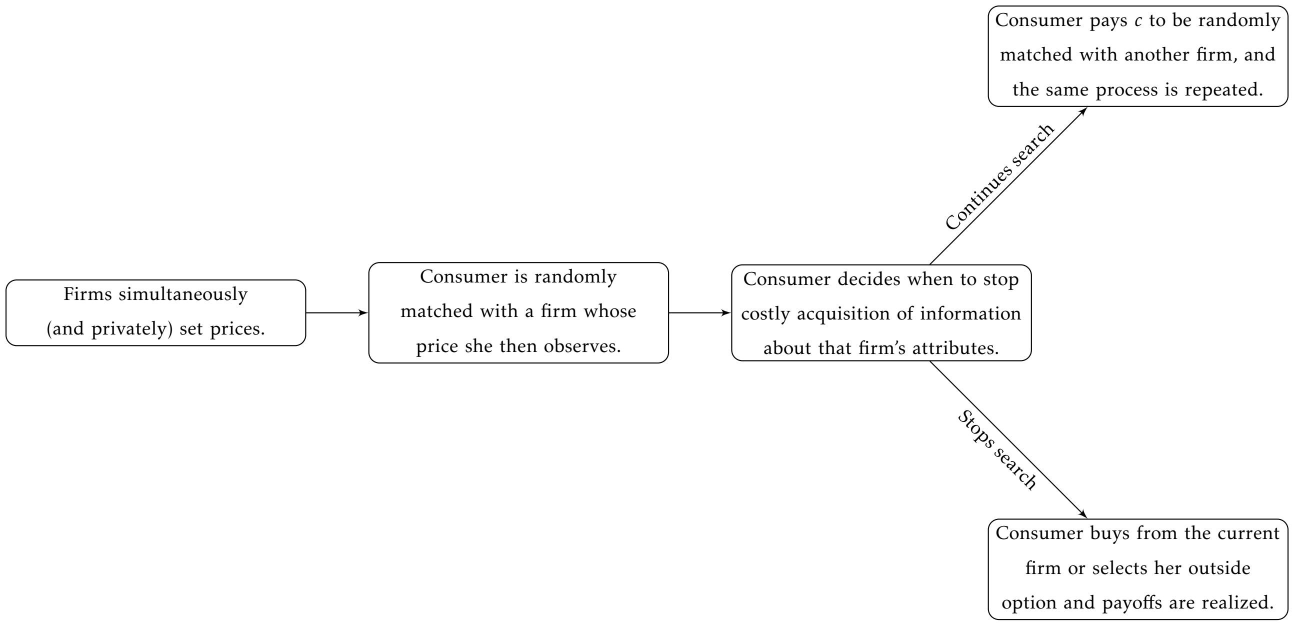


Figure 1: Timing

We denote by $\mathcal{M}(\mu)$ the set of (Borel) probability measures on $\mathbb{R}$ with mean μ and define the *ex ante* cost of distribution $Q \in \mathcal{M}(\mu)$ over values to be the minimal cost at which the consumer can generate Q:

$$C(Q) := \inf_{\tau : F_\tau = Q} \mathbb{E}[\gamma\tau] \,.$$

Then, the results of, e.g., Ankirchner et al. (2015) imply the following remark:

Remark 2.1. The *ex ante* cost of distribution $Q \in \mathcal{M}(\mu)$ over values, $C : \mathcal{M}(\mu) \to \mathbb{R}^+$, is

$$C(Q) = \kappa \int_{-\infty}^{\infty} (x-\mu)^2 \, dQ(x) \,, \quad \text{where} \quad \kappa := \frac{\gamma}{\sigma^2} \,.$$

The precise formula follows via Ito's lemma: for value v_τ,

$$c(v_\tau) = c(\mu) + \int_0^\tau \sigma c'(v_t) \, dW_t + \frac{1}{2} \int_0^\tau \sigma^2 c''(v_t) \, dt \,,$$

where $c(v) := \kappa (v-\mu)^2$. Because τ is a.s. finite, taking expectations, the first term on the right side of the equal sign is 0, and the middle term, being a martingale, vanishes.

Accordingly, instead of solving a consumer's dynamic information acquisition problem at each firm, we may instead solve the following static problem. Let $g(v)$ be a consumer's payoff from stopping her information acquisition at value v. She solves

$$\max_{Q \in \mathcal{M}(\mu)} \left\{ \mathbb{E}_Q[g(v)] - C(Q) \right\} = \max_{Q \in \mathcal{M}(\mu)} \left\{ \int_{-\infty}^{\infty} g(v) dQ(v) - \kappa \int_{-\infty}^{\infty} (v-\mu)^2 \, dQ(v) \right\} \,. \qquad (\star)$$

3 A Consumer's Behavior

Suppose that a consumer conjectures each firm's price to be p.[8] The first step is to determine a consumer's optimal search and information acquisition protocol. Since a consumer faces a stationary environment, her optimal protocol can be calculated recursively. Depending on the search cost c, the information acquisition cost κ, and the *ex ante* expected value of a firm $\mu - p$, three distinct possibilities arise. In the first, there is active search: a consumer learns at a firm and either buys from it or moves on to the next firm. In the other two, there is no active search: a consumer either chooses the outside option without learning, or learns at the first firm and chooses between it and the outside option.

To determine parameters under which each protocol is optimal, as well as a consumer's value Φ from the optimal protocol, we begin by analyzing how she optimally acquires information at a firm when she has the value $a \geq 0$ "in hand." a is a consumer's highest attainable payoff if she does not purchase from the current firm: it takes value $\Phi - c$ in an equilibrium with active search, and 0 in other equilibria.

Given a, optimal learning is given by the concavification[9] of the function specifying the consumer's payoff from a posterior belief v about the firm's value:

$$V(v) = \max\{a, v-p\} - \kappa(v-\mu)^2 = \begin{cases} a - \kappa(v-\mu)^2, & v - p \leq a \\ v - p - \kappa(v-\mu)^2, & v - p \geq a \end{cases}.$$

An example of V and its concavification are depicted in Figure 2. The concavification of V is pinned down by two posterior beliefs, v_L and v_H. If the prior belief μ is between these two beliefs, then they are the support of the optimal posterior distribution. Equivalently, a consumer observes process W until its value hits either v_L or v_H at which point she stops acquiring information. She either become optimistic enough about the firm that she

[8] If firms randomize over prices according to some distribution H, the $g(\cdot)$ from the consumer's objective, $\star$, is $g(v) = \int_{\underline{p}}^{v-a} (v-p)\, dH(p) + \int_{v-a}^{\bar{p}} a\, dH(p)$. This occurs in Section 5, when prices are hidden.

[9] This approach is famously used by Aumann et al. (1995) to characterize the limiting value of repeated games with one-sided incomplete information. Kamenica and Gentzkow (2011) provide a recent promulgation of this technique.

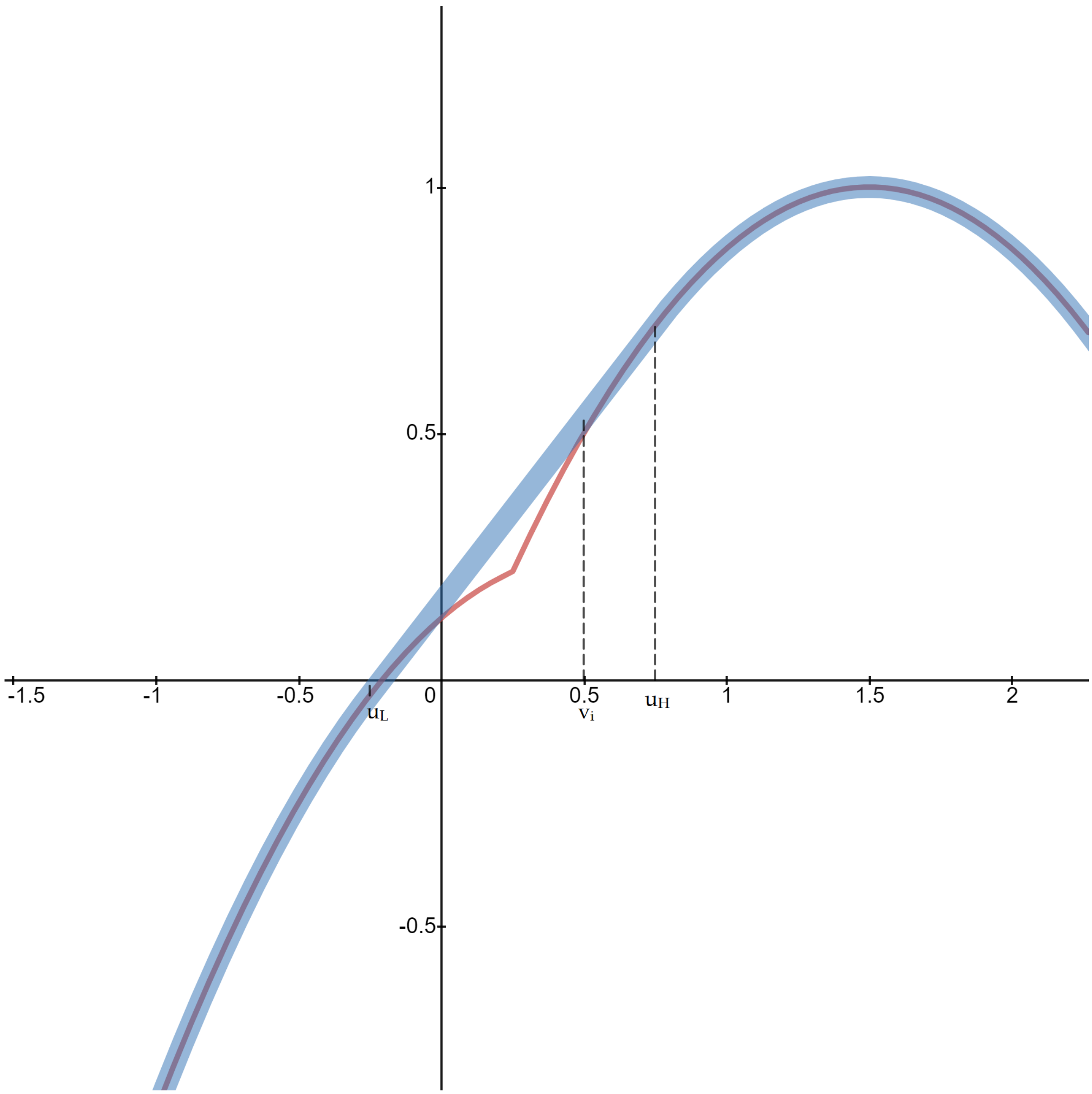


Figure 2: V (in red) and its concavification (in blue) for $a = 1/4$, and $v_i := \mu - p_i = \kappa = 1/2$.

purchases from it, or becomes pessimistic enough that she leaves and gets a. Explicitly,

$$v_L = a + p - \frac{1}{4\kappa}, \quad \text{and} \quad v_H = a + p + \frac{1}{4\kappa}, \quad \text{where} \quad \mathbb{P}(v_H) = \frac{1}{2} + 2\kappa(\mu - p - a) \ .$$

If μ is not between these two posterior beliefs then the consumer does not learn and either purchases the product ($\mu \geq v_H$) or takes her outside option ($\mu \leq v_L$). Predictably, as κ rises, learning occurs over a smaller range of prior values. Furthermore, for higher values of $a + p$, a purchase from the firm without learning is optimal for a smaller range of priors, while walking away without learning is optimal for a larger range of priors.

Denote a consumer's expected payoff from this learning protocol by $V^*(\mu, p; a)$. Then, if the optimal protocol involves active search, a consumer's payoff from it, Φ, is the solution to

$$\Phi = V^*(\mu, p; \Phi - c) \ .$$

Two conditions must hold. The first is that $\Phi - c \geq 0$, since the consumer must (weakly) prefer continuing her search over stopping to take her outside option. This is satisfied if p is low enough. The second is that given $a = \Phi - c$, the prior lies in the learning region. This is satisfied if and only if our assumption that $0 < c \leq 1/(4\kappa)$ holds. Φ_l denotes the optimal value in this case.[10]

Given these observations, we define two terms

$$\Phi_l := \mu - p + c + \frac{1}{4\kappa} - \sqrt{\frac{c}{\kappa}}, \quad \text{and} \quad \Phi_m := \frac{1}{16\kappa} + \frac{\mu - p}{2} + \kappa(\mu - p)^2 \ ,$$

and state the following result.

Proposition 3.1. *If the market price, p, (equivalently, if the information friction, κ) is*

1. *High ($p \geq \mu + 1/(4\kappa)$), there is no trade, and the consumer's payoff is* 0.

2. *Low ($p \leq \mu - \sqrt{c/\kappa} + 1/(4\kappa)$), a consumer both learns and searches and obtains Φ_l.*

3. *In an intermediate region ($\mu - \sqrt{c/\kappa} + 1/(4\kappa) \leq p \leq \mu + 1/(4\kappa)$), a consumer learns at one firm and obtains Φ_m.*

[10]This subscript, l, refers to the "low" (price) region, introduced so as to distinguish this value from the maximal payoffs in other parameter regions. The subscript m denotes "medium," or "moderate."

4 The Firms' Pricing Game

Having pinned down consumers' optimal search and information acquisition protocol for a conjectured price, we may now describe which prices can be sustained in equilibrium. Following much of the literature, we assume that each firm's price is hidden and can only be discovered by a consumer when she visits the firm. Thus, when evaluating the profitability of a deviation, a firm holds consumers' conjecture fixed. It is easy to see that for a low price p (that lies in a consumer's active-search region), we have an equilibrium if p is the optimal price of a monopolist facing a consumer with an outside option $a = \Phi_l - c$. For a medium or high p (that lies in the no-search or no-trade regions), we have an equilibrium if p is the optimal price for a monopolist facing a consumer with an outside option $a = 0$.

Therefore, we begin by characterizing a monopolist's optimal pricing policy for any $a \geq 0$. This is precisely the problem studied in BSV for $a = 0$, and it is easy to modify it. When the monopolist sets a price within the learning region, it faces demand $\mathbb{P}(v_H)$. Outside of this region, demand is either 1 (low price) or 0 (high price). Accordingly, the monopolist's profit maximizing problem yields an optimal price

$$p_M = \begin{cases} \mu - \frac{1}{4\kappa} - a, & \mu \geq \frac{3}{4\kappa} + a \\ \frac{1}{8\kappa} + \frac{\mu - a}{2}, & \frac{3}{4\kappa} + a \geq \mu \geq a - \frac{1}{4\kappa} \end{cases}.$$

This finding is quite intuitive: if the expected quality (net of the outside option), $\mu - a$, is sufficiently high; or, equivalently, information frictions are sufficiently high, the monopolist prices so as to dissuade learning. If $\mu - a$ is in a middle region, or κ is sufficiently low, the monopolist wants to encourage learning. In fact, as κ becomes increasingly small, the monopolist prices ever higher, to take advantage of the consumers who learn that they "absolutely must have the product."

Given this, it is straightforward to characterize the market equilibrium.

Proposition 4.1. *If the level of information frictions, κ, is high compared to the prior expected quality of the product, μ ($\mu \leq -1/(4\kappa)$), no equilibria with trade exist. If the level of information frictions and the search cost, c, are both low compared to the prior expected quality of the* *product ($2\sqrt{c/\kappa} - 1/(4\kappa) \leq \mu$), the unique equilibrium is one in which consumers both search and acquire information. The equilibrium price is $p_l := \sqrt{c/\kappa}$. If the prior expected quality and the level of information frictions are moderate and the search cost is high ($-1/(4\kappa) \leq \mu \leq 2\sqrt{c/\kappa} - 1/(4\kappa)$), the unique equilibrium is one in which consumers visit only one firm and learn there (but do not search actively). The equilibrium price is $p_m = \mu/2 + 1/(8\kappa)$.*

When there is search and information acquisition, a consumer's payoff, a firm's profit, and a consumer's probability of stopping at the current firm are, respectively,

$$\Phi_l^* = \mu + \frac{1}{4\kappa} + c - 2\sqrt{\frac{c}{\kappa}}, \quad \Pi_l^* = 2c, \quad \text{and} \quad \mathbb{P}(v_H) = \frac{1}{2} + 2\kappa(\mu - p - a) = 2\sqrt{c\kappa}\ .$$

When there is only information acquisition (but no search), a consumer's payoff, and a firm's profit are, respectively,

$$\Phi_m^* = \frac{(1+4\kappa\mu)^2}{64\kappa}, \quad \text{and} \quad \Pi_m^* = \frac{(1+4\kappa\mu)^2}{32\kappa}\ .$$

One noteworthy aspect of this result is that there exist no equilibria in which consumers neither search nor acquire information (except in the limiting case when information frictions explode). This stands in sharp contrast to the monopolist scenario (BSV, Pease (2018)), in which the monopolist may wish to set a price so low that the consumer buys without learning. This cannot happen in the search market, since lower prices increase the consumer's outside option (her continuation value), thereby encouraging both learning and search. Related to this is that when there is search and information acquisition, a consumer purchases from *some firm* with probability 1, whereas she settles for her outside option with strictly positive probability when there is no search.

We may also relate our findings to the classical consumer search results without flexible information acquisition. Indeed, keep in mind that we have assumed that $c \leq 1/(4\kappa)$– that neither search cost nor information cost is too high. If this does not hold, just like when search frictions are too high in Wolinsky (1986), the market collapses and we are in the monopoly scenario: consumers visit just one firm. Once above this threshold, the size of the search friction is irrelevant and it is only the magnitude of the information friction that determines prices, profits, and consumer welfare. In the limit, as information frictions explode ($\kappa \to \infty$), we obtain the Diamond paradox (Diamond (1971)): each firm sells with probability one (provided $\mu \geq 0$) and extracts all expected surplus.

4.1 Comparative Statics

At last, we can tackle some of the questions raised in the introduction. First, we observe the unsurprising result that an increase in the prior expected quality of the good, μ, improves welfare in a Pareto sense. Naturally, prices are also increasing in μ.

Proposition 4.2. *Increasing the prior expected quality, μ, is Pareto improving, possibly strictly. If μ is sufficiently high (so there is active search), the market price and a firm's payoff are unaffected by μ, but a consumer's payoff is strictly increasing in μ. If μ is moderate (so there is no active search), prices, profits and consumer payoffs are all strictly increasing in μ.*

Second, we find that an increase in the explicit search cost, c, harms consumers but benefits firms.

Proposition 4.3. *If the search cost, c, is sufficiently small (so there is active search), the market price and a firm's payoff are strictly increasing and a consumer's payoff is strictly decreasing in c. Otherwise (when there is no active search), the market price, a firm's payoff and a consumer's payoff are all unchanging in c.*

It is easy to see that firms require explicit search costs in order to make profits. Indeed, observe that the lower posterior belief, v_L, equals $\mu - \sqrt{c/\kappa}$, which is decreasing in c and diminishes to μ as c vanishes. As costs vanish, the willingness of the consumer to depart for a different firm increases, eroding the rents that each firm can extract.

The effect of a change in κ is more subtle.

Proposition 4.4. *Prices are strictly decreasing in the information cost, κ. If κ is sufficiently small, welfare is Pareto decreasing in κ. If μ is sufficiently large (or c is sufficiently small) and κ is sufficiently large, welfare is Pareto increasing in κ, possibly strictly (if the parameters are such that there is no active search).*

Notably, it is possible that an increase in information frictions can be Pareto improving. The effect of these frictions on the price seems natural: notice that when κ is higher, the demand $\mathbb{P}(v_H)$ is more responsive to changes in p, holding the continuation value fixed–a firm's "demand curve" is steeper. Consequently, at equilibrium, the solution of the optimization problem of a firm entails lowering the price slightly as a reaction to an increase in κ. Moreover, as it turns out, this balance is perfect: in regions of the parameter space with active search, a firm's profit is unaffected by changes in κ since, as noted above, the rents it can extract are limited by the explicit search cost.

In contrast, a change in the information acquisition cost affects a consumer's welfare in two ways. *Ceteris paribus*, an increase in κ hurts consumers, as information is more difficult to obtain. However, it also results in a decrease in the market price, which is to a consumer's benefit. The overall effect of κ on consumer welfare; therefore, is determined by which of these two forces dominates. When κ is low (and therefore prices are high) the effect of an increase in κ on the price is too small to outweigh the negative direct affect on learning. Conversely, a high κ flips the relationship.

In the monopolists' problems of BSV and Pease (2018), prices also decrease in the cost of information (for that region in which the monopolist does not simply want to price sufficiently low as to discourage information acquisition). Moreover, BSV also find that increased information frictions may lead to a Pareto improvement in market welfare. We wish to briefly argue that it is not obvious that these relationships should continue to hold in a market with search frictions. Indeed, observe that in contrast to the monopoly setting, in which information frictions affect the monopolist's pricing decision only through their effect on the consumer's information acquisition protocol; in our model, information frictions also affect the pricing behavior of the other firms in the market, which alters the consumer's outside option. A larger outside option depresses prices; and so when the (equilibrium) consumer welfare is increasing in κ, the two effects act in synergy, so the decrease in equilibrium price is predictable. However, when the (equilibrium) consumer welfare is decreasing in κ, the two forces conflict, so our finding that the market price, nevertheless, is decreasing in the cost of information may not be apparent *a priori*.

Proceeding onward, it is easy to see that if there is active search, the expected search duration is $1/(2\sqrt{c\kappa})$, which is predictably decreasing in c and κ. If there is no active search, the expected search duration is just 1.

Proposition 4.5. *(Expected) search duration as a function of the prior expected quality, μ, is increasing in μ: it is a step function with one discontinuity at $\mu = 2\sqrt{c/\kappa} - 1/(4\kappa)$, where it jumps* *up from* 1 *to* $1/(2\sqrt{c\kappa})$. *Search duration is decreasing in the search cost, c: it is strictly decreasing in c for c sufficiently small (in the active search region) and constant thereafter (in the region without active search). Search duration may be non-monotone in the information cost, κ. In particular, for a region of the parameter space, there is active search for κ sufficiently low or sufficiently high, but no active search in an intermediate region.*

Figure 3 illustrates the results of Propositions 4.4 and 4.5. In the proof of the former (in Appendix A.5) we find that the parameter space (for $\mu > 0$) can be divided into three cases, each of which is depicted in the figure. The portions of the domain in which Π is flat are those regions of κ (for the fixed κ and μ) that facilitate active search. Note that the graph exhibits considerable non-monotonicity in κ: over different intervals of κ profit and consumer welfare may both be increasing or decreasing. Moreover, the equilibrium may involve active search for small and large κ, but not for moderate κ. Accordingly, the expected search duration may have discrete jumps both up and down.

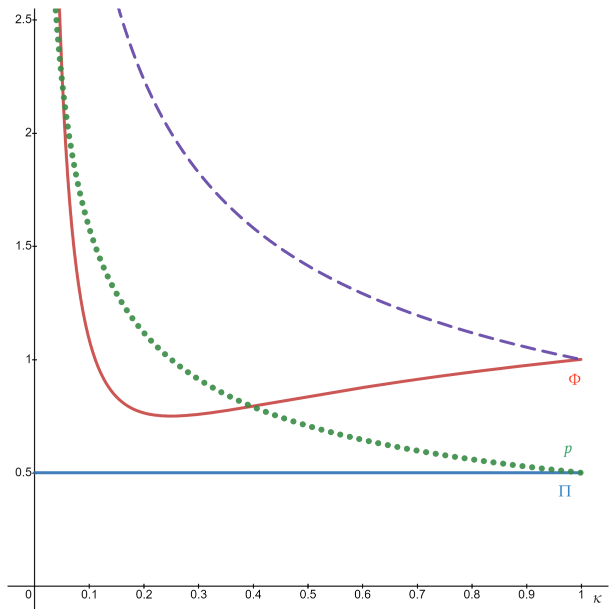
4.2 Obfuscation

By now, a sizeable collection of papers on obfuscation in search markets has emerged. These papers, both theoretical[11] and empirical[12] explore how firms may strategically increase consumers' search costs to allow for higher mark-ups. Instead of firms obfuscating by increasing search costs (the typical type of obfuscation explored in the literature), a different form of obfuscation suggests itself in our paper. Namely, we may allow each firm to not only set a price but also choose a level of information friction κ from some interval of possible frictions $[\underline{\kappa}, \bar{\kappa}]$, where $\underline{\kappa} > 0$ and $\bar{\kappa} \leq \infty$.

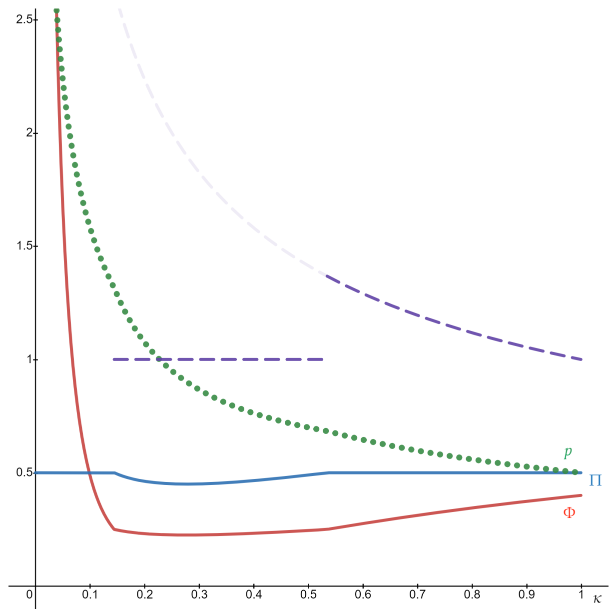
How might firms be able to choose κ? Recall that in a consumer's dynamic problem, she observes in sequence each attribute, which is a symmetric (binary) random variable. By adding noise to this random variable, a firm would decrease the variance of the corresponding Brownian process, thereby increasing κ. Firms could also increase the explicit cost of inspecting each attribute, γ, which also raises κ.

[11] See, e.g., Wolinsky (1986), Gu and Wenzel (2014), Hämäläinen (2018), and Petrikaitė (2018).

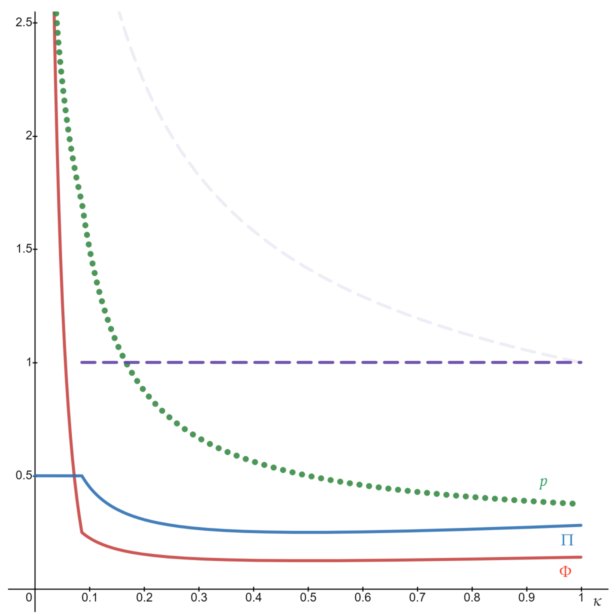
[12] See, e.g., Ellison and Ellison (2009) and Richards et al. (2019). In addition, Ellison (2016) discusses obfuscation in her survey of price search.



(a) **Case 1:** $\mu > 0$ and $c \leq \mu/4$.



(b) **Case 2:** $\mu > 0$ and $\mu/3 \geq c \geq \mu/4$.



(c) **Case 3:** $\mu > 0$ and $c \geq \mu/3$.

Figure 3: Consumer welfare (solid red), profit (solid blue), price (dotted green), and expected search duration (dashed purple) as functions of the information cost, κ.

A pair (κ, p) is an equilibrium provided it is the solution to a firm's monopoly problem with a determined by the equilibrium. It is easy to see that the equilibrium κ must be a corner solution. *Viz.*,

Remark 4.6. If firms may choose both price, p, and information cost, κ, the equilibrium $\kappa^* \in \{\underline{\kappa}, \bar{\kappa}\}$.

It is straightforward, though rather tedious to characterize the explicit equilibria of this game, but we believe the main insight we discover regarding obfuscation is contained in the remark and so do not do so. As the remark states, when firms may strategically obfuscate the product value by hindering the observability of a product's attributes, they either muddle this process as much as possible or as minimally as possible. An interior level of obfuscation never constitutes an equilibrium.

5 Hidden Prices

So far, we have assumed that when a consumer visits a firm, she observes the price for the item *before* acquiring information. This seems like reasonable assumption for many sorts of consumer goods like, e.g., shoes, bicycles, perfumes, headphones, foodstuffs. However, there are other markets in which it may be more natural to assume that a consumer acquires information about a product before seeing its price. One way this may manifest is when firms include hidden fees or charges that can only be observed later on in the purchase process.

In this section, we analyze the situation in which a consumer may not observe a firm's price until after she has acquired information at the firm.[13] As we will shortly see, this changes the equilibrium behavior significantly: there is now an informational hold-up problem, which any equilibrium must accommodate.

We begin by stating the outcome when there is a single (monopolist) firm.

[13] Importantly, each firm sets its price in advance and may not observe consumers' behavior beforehand.

5.1 Monopoly With Hidden Prices

As we did in the observable prices case, we stipulate that a consumer has an outside option of $a \geq 0$. In contrast to the observable prices scenario, we find that the existence of a pure strategy pricing equilibrium for the monopolist is incompatible with trade.

Lemma 5.1. *There are equilibria in which the monopolist chooses a deterministic price if and only if the prior expected quality is below the value of the outside option ($\mu \leq a$). In any such pure-strategy equilibrium, there is no trade.*

If the monopolist mixes over prices, it must be indifferent over all prices in the support of her mixture. That is, given a consumer's distribution over posteriors, F, the monopolist's profit is given by

$$\hat{\Pi}(p) = p(1 - F(p + a)) \ ,$$

which equals some constant $\lambda > 0$ for all p in the support of her mixed strategy.

Theorem 5.2. *If the prior expected quality is strictly higher than the outside option $\mu > a$, there is an essentially unique*[14] *equilibrium in the monopolist scenario. The empirical distribution over consumer valuations is a truncated Pareto distribution, and the monopolist chooses a uniform distribution over prices.*

The explicit expressions for the distribution over consumer valuations, F, and the monopolist's distribution over prices, G, are

$$F(x) := 1 - \frac{\underline{x}_M}{x}, \quad \text{on} \quad [\underline{x}_M, \bar{x}_M] \ ,$$

[14] That is, any equilibrium must be such that the empirical distribution over valuations is F. One way this could be obtained is if each consumer randomized and chose distribution F herself. Because there is a unit mass of consumers, one might worry about whether one could construct a continuum of independent individual random variables that yields the desired empirical distribution over values by the Exact Law of Large Numbers. Sun (2006) allows for precisely such a construction to be done rigorously. Alternatively, we could assume that there is a random variable $Y \sim H$ with support $[0,1]$, which is a consumer's label. By, e.g., Winkler (1988) there is a distribution H such that a consumer with label y (deterministically) chooses a binary distribution over values with support $\{v_L(y), v_H(y)\}$ such that the resulting compound distribution is precisely F. Yet another approach would be to assume that there is a single representative consumer, who herself mixes over values according to F.

and

$$G(p) := 2\kappa\left(p - \underline{p}_M\right), \quad \text{on} \quad \left[\underline{p}_M, \bar{p}_M\right],$$

where $\underline{p}_M$ solves

$$\ln\left\{1 + \frac{1}{2\kappa\underline{p}_M}\right\} = \frac{\mu - a}{\underline{p}_M} - 1,$$

and $\bar{p}_M = \underline{p}_M + 1/(2\kappa)$, and $\underline{x}_M = \underline{p}_M + a$. The equilibrium vector of payoffs is

$$\hat{\Pi}_M^* = \underline{p}_M, \quad \text{and} \quad \hat{\Phi}_M^* = \kappa\left(\mu - \underline{x}_M\right)^2 + a.$$

The crucial feature of the market's distribution over valuations, F, is that it begets unit elastic demand for the monopolist, who is, therefore, willing to mix over prices. The monopolist chooses a distribution over prices that yields a payoff for consumers (net of information acquisition costs) that is an affine function of the realized posterior values. They, in turn, are therefore willing to choose any distribution with support on $[\underline{x}_M, \bar{x}_M]$. As the concavification approach suggests, if a consumer's payoff were strictly convex in her value, a binary distribution would be uniquely optimal. Conversely, if her payoff were strictly concave in her value, she would not acquire any information.

5.1.1 Monopoly Comparative Statics

Just as how the equilibrium price when prices are observable increases in μ and κ, so does the lower bound for its distribution over prices when prices are hidden.

Lemma 5.3. *The lower bound for the monopoly price, $\underline{p}_M$, is strictly increasing in the information cost, κ, and the prior expected quality, μ.*

However, because the monopolist's profit is precisely this lower bound, $\underline{p}_M$, information frictions are unambiguously good for the monopolist when prices are hidden. In the limit, as κ explodes, $\underline{p}_M$ and $\bar{p}_M$ converge to $\mu - a$ and so the monopolist leaves consumers with zero rents. In contrast to the observable prices case–when consumer welfare is initially decreasing in κ, then increasing, then decreasing again (after entering the region in which the monopolist does not induce learning)–when prices are hidden, consumer welfare is initially increasing in κ then decreasing. In the limit, as κ explodes, both profit and consumer welfare are the same regardless of when consumers may observe the price.

Lemma 5.4. *The monopolist's profit is strictly increasing in the information cost, κ, and the prior expected quality, μ. Consumer welfare is strictly increasing in μ and is (strictly) increasing in κ if and only if κ is sufficiently low ($\underline{p}_M \kappa \lesssim .337$).*

5.2 The Monopolistic Competition Equilibrium

If there exists an equilibrium with learning and active search, the equilibrium is analogous to that given in the monopolist scenario, where we substitute a consumer's continuation value, $\hat{\Phi} - c$, in for a:

Theorem 5.5. *If the prior expected quality, μ, is strictly greater than the value of the outside option, 0, there is an essentially unique equilibrium in the monopolistic competition scenario. The empirical distribution over consumer valuations is a truncated Pareto distribution, and the monopolist chooses a uniform distribution over prices.*

Importantly, the equilibrium involves active search if and only if $\mu - \sqrt{c/\kappa} > \underline{p}$, where the price $\underline{p}$ solves

$$\ln\left\{1 + \frac{1}{2\kappa \underline{p}}\right\} = \frac{1}{\underline{p}}\sqrt{\frac{c}{\kappa}}\,.$$

When there is active search, the explicit expressions for the empirical distribution over consumer valuations, F, and the firms' distributions over prices, G, are

$$F(x) := 1 - \frac{\mu - \sqrt{\frac{c}{\kappa}}}{x}, \quad \text{on} \quad \left[\mu - \sqrt{\frac{c}{\kappa}}, \mu - \sqrt{\frac{c}{\kappa}} + \frac{1}{2\kappa}\right],$$

and

$$G(p) := 2\kappa\left(p - \underline{p}\right), \quad \text{on} \quad \left[\underline{p}, \bar{p}\right].$$

The equilibrium vector of payoffs is

$$\hat{\Pi} = \underline{p}, \quad \text{and} \quad \hat{\Phi} = \mu - \underline{p} + c - \sqrt{\frac{c}{\kappa}}\,.$$

If $\mu - \sqrt{c/\kappa} \leq \underline{p}$, so there is no active search, the unique equilibrium is given in Theorem 5.2, with $a = 0$.

Because $\underline{p}$ is only given implicitly, the condition that demarcates the active search equilibrium is difficult to parse (since it itself is in terms of the equilibrium object $\underline{p}$). As a result, it is helpful to supplement the theorem with the following proposition.

Proposition 5.6. *For any search cost, c, there is a threshold prior expected quality, $\tilde{\mu}(c) > 0$, such that if $\mu < \tilde{\mu}(c)$ the equilibrium does not involve active search irrespective of the information cost, κ. If $\mu > \tilde{\mu}(c)$, there exist $\underline{\kappa}(c, \mu) > 0$ and $\bar{\kappa}(c, \mu) < 1/(4c)$ (with $\bar{\kappa} > \underline{\kappa}$) such that the equilibrium involves active search if and only if $\kappa \in [\underline{\kappa}, \bar{\kappa}]$.*

Thus, we see that either the prior expected value for the product, μ, is so low that there is no active search, regardless of the information cost, κ; or it is sufficiently high that there is active search if and only if κ lies in an intermediate region.

There are a number of interesting implications of this equilibrium. Despite the homogeneity of the market, and even though search is not directed, the unique equilibrium exhibits price dispersion. Furthermore, there may also be significant diversity of consumer behavior. Namely, the empirical distribution over consumer valuations may be driven by varying levels of learning by different consumers: some learn a lot at each firm while others learn very little. The unique equilibrium; therefore, justifies a broad spectrum of different (ostensibly capricious) behavior by identical consumers.

5.2.1 Monopolistic Competition Comparative Statics

Proposition 5.7. *The lower bound for the price, $\underline{p}$, is strictly increasing in the search cost, c. In the active-search region (if it exists), $\underline{p}$ is independent of the prior expected quality, μ, and is (strictly) decreasing in the information cost, κ, if and only if $h(c, \kappa) := \ln\left(\sqrt{c\kappa}\right) + 2 - 2\sqrt{c\kappa} \leq$ $(<)$ 0. Outside of the active-search region, $\underline{p}$ is strictly increasing in κ and μ.*

A simple calculation (or a glance at its graph) reveals that h is strictly increasing in κ. Moreover, $\lim_{\kappa \searrow 0} h = -\infty$ and $h(1/(4\kappa)) = \ln(1/2) + 1 > 0$, so h has a unique root at some $\tilde{\kappa} \in (0, 1/(4c))$.

Because a firm's profit is merely the lower bound for the support of the price distribution, the following result is evident.

Corollary 5.8. *A firm's profit is strictly increasing in the search cost, c. In the active-search region, a firm's profit is increasing in the prior expected quality, μ, and is (strictly) decreasing in the information cost, κ, if and only if $h(c, \kappa) \leq (<)$ 0. Outside of the active-search region, profits are strictly increasing in κ and μ.*

A direct calculation reveals that a consumer's welfare is strictly decreasing in c. On the other hand, note that

$$\hat{\Phi}'(\kappa) = -\underline{p}'(\kappa) + \frac{1}{2\kappa}\sqrt{\frac{c}{\kappa}},$$

and so if the price is decreasing in κ, a consumer's welfare is increasing in κ. Naturally, even if the price is increasing in κ, a consumer's welfare may still be increasing in κ, provided the price increase is not too large. Summing things up,

Proposition 5.9. *A consumer's payoff, $\hat{\Phi}$, is strictly decreasing in the search cost, c. In the active-search region, $\hat{\Phi}$ is strictly increasing in the prior expected quality, μ, and is (strictly) increasing in the information cost, κ, if and only if κ $(<) \leq \kappa'$, where $\kappa' \in (\tilde{\kappa}, 1/(4c))$. Outside of the active search region, a consumer's payoff is (strictly) increasing in κ if and only if κ is sufficiently low.*

6 Who (if Anyone) Benefits From Hidden Prices?

One interpretation of the "hidden prices" version of the model is as a market in which firms can introduce hidden fees late in the purchase. This issue has come to the forefront of recent policy debates; and in the United States of America alone, a number of bills have been proposed, or rules imposed, recently to restrict drip pricing and other forms of hidden (additional) prices.[15] Our paper; therefore, can provide another perspective on this debate, since we may compare the various market outcomes (profits, consumer welfare, and prices) when prices are observable before learning versus when they are not.

A priori one might suppose that hidden prices are to consumers' detriment, since it affects their learning adversely–a consumer understands that learning that she loves the product may allow the firm to charge a high price to exploit this enthusiasm. Further-

[15]In their background section, Santana et al. (2020) include a nice discussion of such recent rules and proposed legislation. Additional papers with similar analyses of the effect of hidden fees and prices on consumer shopping behavior include Blake et al. (2018) and Bradley and Feldman (2020). The modal explanation for the adverse effects of hidden fees is behavioral: and in particular price (and tax) salience is often cited. In contrast, hidden prices affect behavior in our model strategically–our consumers are fully rational and understand the ramifications of the hold-up problem in which they find themselves.

more, it also removes the ability of consumers to react to price changes and thereby discipline firms. As we shortly discover, this prediction is correct: consumers prefer observable fees, both when there is a search market as well as when there is a single monopolistic seller of the product. It is less clear which information regime firms prefer. On one hand, firms might have more impunity to set higher prices since consumers have less flexibility to react. However, consumers are strategic and so may not learn as much as the firms would like when prices are hidden.

Our next proposition sums up the stark outcome when there is a monopolist. This result is predictable. By allowing the monopolist the power to commit to its price, consumers have more incentive to learn, since they know that they will not be exploited. Figure 4 depicts the monopolist's profits with observable prices and hidden prices as well as its price when prices are observable and the lower bound of its distribution over prices when prices are hidden.

Proposition 6.1. *Both monopolist and consumers prefer prices to be observable before learning.*

As we stated above, consumers also prefer to observe prices before learning in the search market with multiple firms. The intuition is analogous to that when there is a single seller: consumers no longer have to worry about being exploited and can therefore learn optimally as a result.

Proposition 6.2. *Consumer welfare when prices are observed before learning is strictly higher than when prices are observed after learning.*

Surprisingly, the result from the monopoly scenario does not always carry over for firms. That is, it is possible that firms may actually prefer hidden prices. This occurs when μ and κ are sufficiently high (in comparison to c). The explanation for this finding is as follows: if the parameters are such that there is active search when prices are hidden, there must be active search when prices are observed. Moreover, recall that when there is active search, a firm's profit is independent of both the prior quality and the level of information frictions and is determined only by c, which is quite low by assumption. In contrast, when prices are hidden (and there is active search), c must be so low (or μ and κ so high) that the lower bound of the equilibrium price, $\underline{p}$ is greater than $2c$, the profit

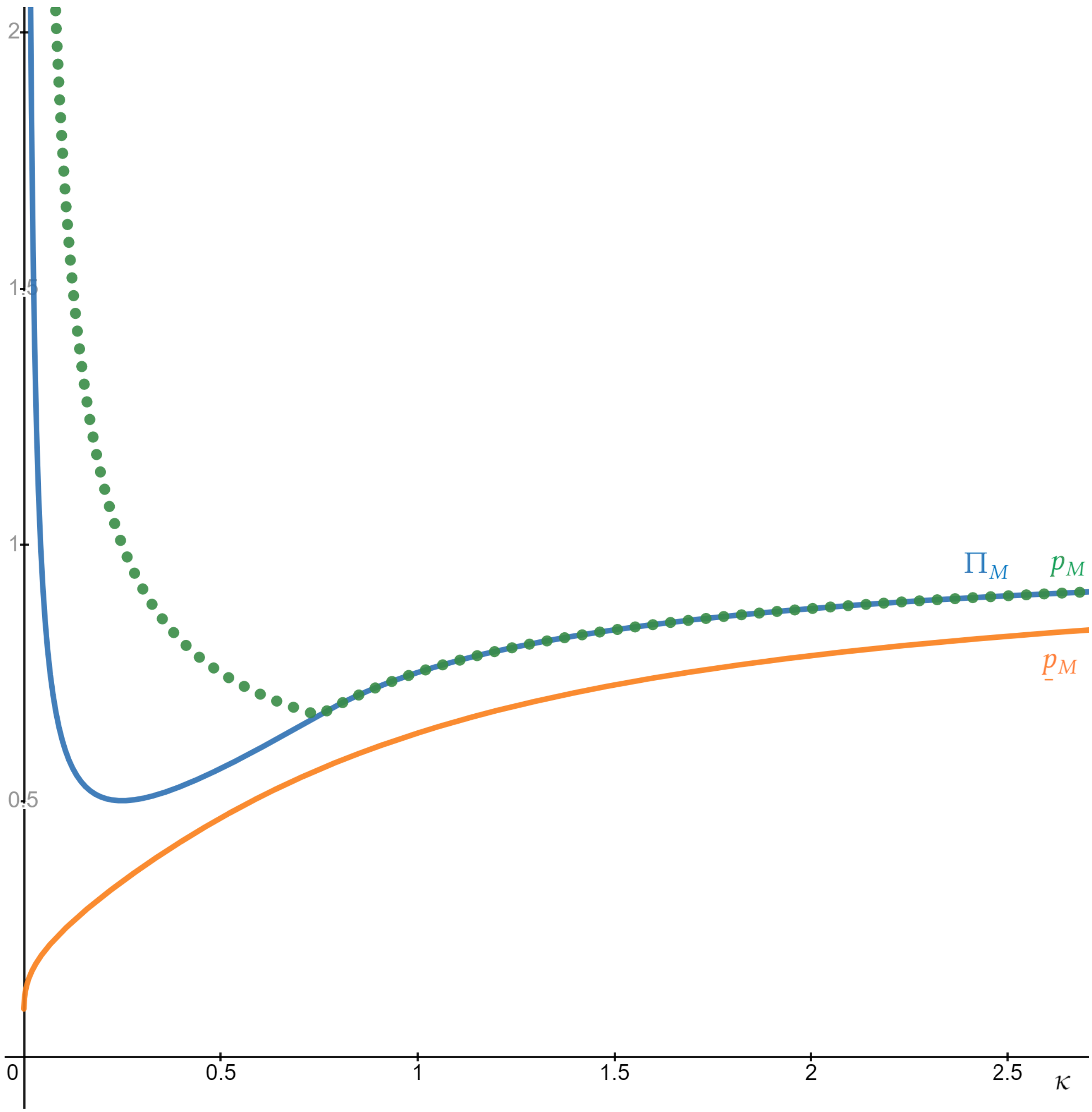


Figure 4: Monopolist's profit with observable prices (solid blue), monopolist price with observable prices (dotted green), and monopolist's profit and price lower bound with hidden prices (solid orange) as functions of the information cost, κ.

in the observable prices set-up. Of course, $\underline{p}$ is precisely a firm's profit when prices are hidden, and so its profits are, therefore, higher.

When the parameters are such that consumers do not actively search in either observation regime, we are back in the monopolist setting and so observable prices are preferred by the firms. The third and final possibility is that the equilibrium involves active search if and only if prices are observable. This is a mix of the other two cases: if μ and κ are sufficiently high, firms prefer hidden prices, but not if they are too low.

Proposition 6.3. *Firms prefer that consumers observe prices before learning if and only if the prior expected quality, μ, is sufficiently low.*

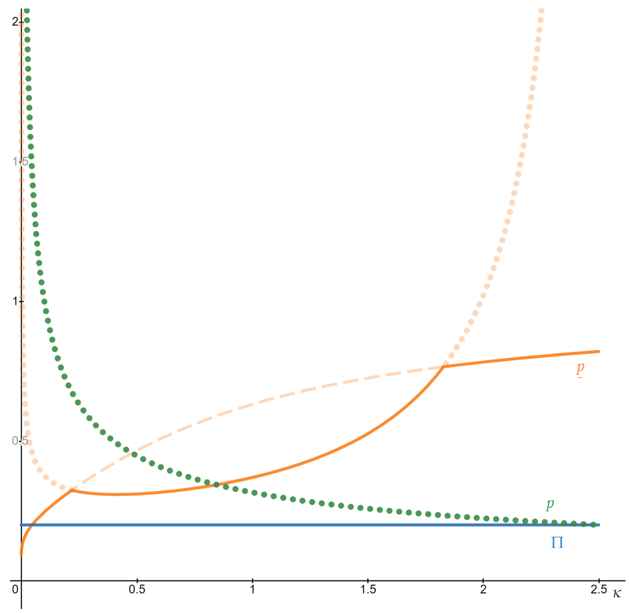
Figure 5 depicts firms' profits with observable prices and hidden prices as well as their prices when prices are observable and the lower bound of the market distribution over prices when prices are hidden.

7 Discussion

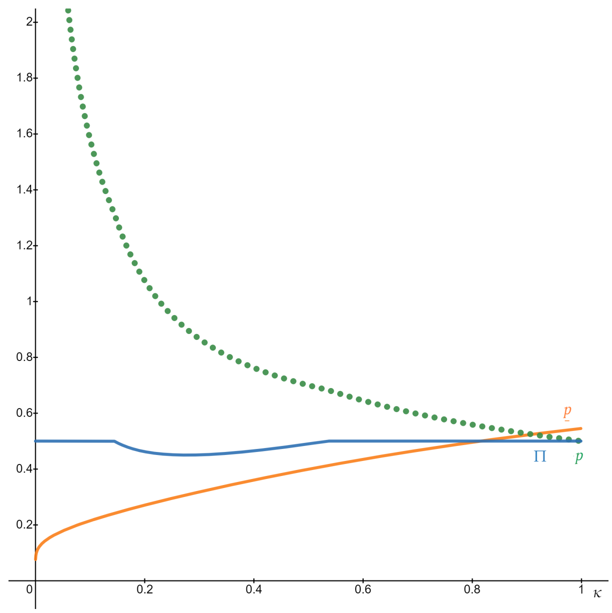
With the exception of Guo (2021), the existing literature that explores sequential consumer search does not allow for flexible information acquisition by consumers. Similarly, the existing rational inattention literature does not tackle the dynamic problem of consumers searching and flexibly acquiring information in sequence. We formulate and solve a model that incorporates these aspects, which allows us to identify and highlight the vital differences between search and information frictions.

Despite the ability of consumers to adapt their learning to the prices they observe, higher search frictions benefit firms and lead to greater market prices. Increased search costs make consumers more reluctant to leave firms, making them *less* responsive to price increases, which firms can exploit. In contrast, higher information frictions make consumers *more* responsive to price increases, thereby decreasing market prices and potentially benefiting consumers.

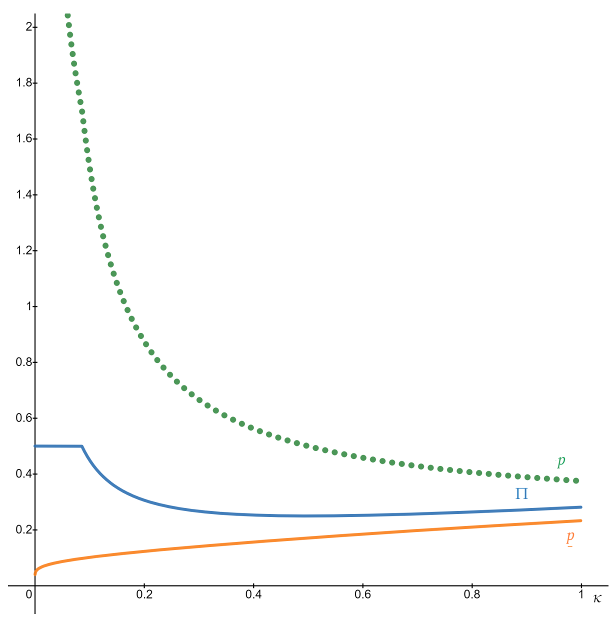
It is important to note that these relationships–that equilibrium prices increase in search frictions (when prices are not advertised) and that increased information frictions



(a) **Case 1:** $\mu > 0$ and $c \leq \mu/4$.



(b) **Case 2:** $\mu > 0$ and $\mu/3 \geq c \geq \mu/4$.



(c) **Case 3:** $\mu > 0$ and $c \geq \mu/3$.

Figure 5: Profit with observable prices (solid blue), price with observable prices (dotted green), and profit and price lower bound with hidden prices (solid orange) as functions of κ.

lead to lower prices and potential welfare increases–feature in many (or at least several) of the papers that focus on a single of our two dimensions, i.e., those that look at a monopolist selling to consumers who may acquire information or at oligopolistic markets with search frictions in which consumers may not control their learning explicitly. By combining these two elements in a single unified framework, we explore how they interact and are able to illustrate cleanly how and why the two frictions have such different effects on equilibrium outcomes. In addition, we find that these relationships hold (for the most part) even when consumers cannot observe prices before acquiring information: firms benefit at the expense of consumers as search costs increase, whereas an increase in information frictions may improve the lot of consumers.

Our analysis; therefore, suggests that mandated decreases in information frictions (mandatory disclosure rules or transparency requirements, say) may be to the market's detriment. Moreover, our comparison of cases in which a firm's price is observed before or after information acquisition allows us to provide input on policies banning hidden fees. We find that consumers are always better off when they can observe prices before learning but that firms may be hurt by the transparency, in spite of the hold-up problem that ensues when prices are hidden.

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A Omitted Proofs and Derivations

A.1 Section 3 Derivations

We have

$$V^{*}(\mu,p;a)=\begin{cases} a, & \mu\le a+p-\frac{1}{4\kappa} \\ \frac{1}{16\kappa}+\frac{\mu-p+a}{2}+\kappa(\mu-p-a)^{2}, & a+p-\frac{1}{4\kappa}\le\mu\le a+p+\frac{1}{4\kappa} \\ \mu-p, & a+p+\frac{1}{4\kappa}\le\mu \end{cases}.$$

Φ_l is the value of Φ such that

$$\Phi=\frac{1}{16\kappa}+\frac{\mu-p+\Phi-c}{2}+\kappa(\Phi-c-(\mu-p))^{2}\ .$$

The condition $\Phi_l-c\ge 0$ reduces to $p\le\mu-\sqrt{c/\kappa}+1/(4\kappa)$.

A.2 Section 4 Derivations

The profit of a monopolist charging price p to a consumer with outside option $a \geq 0$ is given by

$$\Pi_M(p) = \begin{cases} 0, & p \geq \mu + \frac{1}{4\kappa} - a \\ p\left(\frac{1}{2} + 2\kappa(\mu - p - a)\right), & \mu - \frac{1}{4\kappa} - a \leq p \leq \mu + \frac{1}{4\kappa} - a \\ p, & p \leq \mu - \frac{1}{4\kappa} - a \end{cases}.$$

The monopolist's maximal profit (at the optimal price), Π_M^*, and corresponding consumer payoff, Φ_M^*, are given by

$$\Pi_M^* = \begin{cases} \mu - \frac{1}{4\kappa} - a, & \mu \geq \frac{3}{4\kappa} + a \\ \frac{(1+4\kappa(\mu-a))^2}{32\kappa}, & \frac{3}{4\kappa} + a \geq \mu \geq a - \frac{1}{4\kappa} \\ 0, & \mu \leq a - \frac{1}{4\kappa} \end{cases}, \text{ and } \quad \Phi_M^* = \begin{cases} \frac{1}{4\kappa} + a, & \mu \geq \frac{3}{4\kappa} + a \\ \frac{\Pi_M^*}{2} + a, & \frac{3}{4\kappa} + a \geq \mu \end{cases}.$$

A.3 Proposition 4.2 Proof

Proof. By direct computation, if $2\sqrt{c/\kappa} - 1/(4\kappa) > \mu \geq -1/(4\kappa)$, the market price, a firm's payoff and a consumer's payoff are strictly increasing in μ. Analogously, if $\mu \geq 2\sqrt{c/\kappa} - 1/(4\kappa)$, the market price and a firm's payoff are unaffected by μ, but a consumer's payoff is strictly increasing in μ. ∎

A.4 Proposition 4.3 Proof

Proof. Recall that there is active search and information acquisition in equilibrium provided

$$\mu \geq 2\sqrt{\frac{c}{\kappa}} - \frac{1}{4\kappa}. \tag{A1}$$

Rearranging, we get a number of cases.

Case 1: If $\mu \geq 3/(4\kappa)$, Inequality A1 holds (since $0 < c < 1/(4\kappa)$).

Case 2: If $-1/(4\kappa) \leq \mu < 3/(4\kappa)$, Inequality A1 holds if and only if

$$0 < c < \frac{(1+4\kappa\mu)^2}{64\kappa}.$$

Case 3: If $\mu \le -1/(4\kappa)$, Inequality $A1$ does not hold.

In the active-search region, a simple calculation reveals that Φ_l^* is strictly decreasing in c. In this region, Π_l^* and p_l are strictly increasing in c. Trivially, these equilibrium objects are independent of c in the region without active search. ■

A.5 Proposition 4.4 Proof

Proof. In the active-search region,

$$\frac{\partial \Phi_l^*}{\partial \kappa} = -\frac{1}{4\kappa^2} + \frac{1}{\kappa}\sqrt{\frac{c}{\kappa}} \ge \ (>)\ 0 \quad \Leftrightarrow \quad \kappa \ge \ (>)\ \frac{1}{16c} \ .$$

In this region, Π_l^* is unchanging in κ, and p_l is strictly decreasing in κ. In the region without active search,

$$\frac{\partial \Phi_m^*}{\partial \kappa} = \frac{\mu^2}{4} - \frac{1}{64\kappa^2} \ge \ (>)\ 0 \quad \Leftrightarrow \quad \kappa \ge \ (>)\ \left|\frac{1}{4\mu}\right| \ .$$

Since $\Pi_m^* = 2\Phi_m^*$, the effect of a change of κ on a firm's profit is identical to its effect on consumer welfare in this (no-search) region. p_m is strictly decreasing in κ.

Rearranging Equation $A1$, we observe that there are four cases to exhaust:

Case 1: $\mu > 0$ and $0 < c \le \mu/4$. Here, $\kappa < 1/(4c)$ implies inequality $A1$. Consequently,

$$\operatorname{sgn}\Phi^{*'}(\kappa) = \begin{cases} -1, & 0 < \kappa < \frac{1}{16c} \\ 0, & \kappa = \frac{1}{16c} \\ 1, & \frac{1}{16c} < \kappa < \frac{1}{4c} \end{cases} , \quad \text{and} \quad \operatorname{sgn}\Pi^{*'}(\kappa) = 0 \ .$$

Case 2: $\mu > 0$ and $\mu/4 < c < \mu/3$. Here, $A1$ holds either if

$$0 < \kappa \le \frac{8c - \mu - 4\sqrt{(4c-\mu)\,c}}{4\mu^2} =: \underline{\kappa}, \quad \text{or} \quad \bar{\kappa} := \frac{8c - \mu + 4\sqrt{(4c-\mu)\,c}}{4\mu^2} \le \kappa < \frac{1}{4c} \ .$$

Consequently,

$$\operatorname{sgn}\Phi^{*'}(\kappa)=\begin{cases}-1, & 0<\kappa<\underline{\kappa}\\ -1, & \underline{\kappa}\leq\kappa<\frac{1}{4\mu}\\ 0, & \kappa=\frac{1}{4\mu}\\ 1, & \frac{1}{4\mu}<\kappa<\bar{\kappa}\\ 1, & \bar{\kappa}\leq\kappa<\frac{1}{4c}\end{cases},\quad\text{and}\quad \operatorname{sgn}\Pi^{*'}(\kappa)=\begin{cases}0, & 0<\kappa<\underline{\kappa}\\ -1, & \underline{\kappa}\leq\kappa<\frac{1}{4\mu}\\ 0, & \kappa=\frac{1}{4\mu}\\ 1, & \frac{1}{4\mu}<\kappa<\bar{\kappa}\\ 0, & \bar{\kappa}\leq\kappa<\frac{1}{4c}\end{cases}.$$

Case 3: $\mu\geq 0$ and $c\geq \mu/3$. Here, $A1$ holds if $0<\kappa\leq\underline{\kappa}$. Accordingly,

$$\operatorname{sgn}\Phi^{*'}(\kappa)=\begin{cases}-1, & 0<\kappa<\underline{\kappa}\\ -1, & \underline{\kappa}\leq\kappa<\frac{1}{4\mu}\\ 0, & \kappa=\frac{1}{4\mu}\\ 1, & \frac{1}{4\mu}<\kappa<\frac{1}{4c}\end{cases},\quad\text{and}\quad \operatorname{sgn}\Pi^{*'}(\kappa)=\begin{cases}0, & 0<\kappa<\underline{\kappa}\\ -1, & \underline{\kappa}\leq\kappa<\frac{1}{4\mu}\\ 0, & \kappa=\frac{1}{4\mu}\\ 1, & \frac{1}{4\mu}<\kappa<\frac{1}{4c}\end{cases}.$$

Case 4: $\mu<0$. Again, $A1$ holds if $0<\kappa\leq\underline{\kappa}$. Accordingly,

$$\operatorname{sgn}\Phi^{*'}(\kappa)=\begin{cases}-1, & 0<\kappa<\underline{\kappa}\\ 1, & \underline{\kappa}\leq\kappa<\frac{1}{4c}\end{cases},\quad\text{and}\quad \operatorname{sgn}\Pi^{*'}(\kappa)=\begin{cases}0, & 0<\kappa<\underline{\kappa}\\ 1, & \underline{\kappa}\leq\kappa<\frac{1}{4c}\end{cases}.$$

■

A.6 Lemma 5.1 Proof

Proof. Let us begin by establishing that if $\mu>a$ there exist no pure-strategy equilibria. For convenience, we assume that there is a single representative consumer. Suppose for the sake of contradiction that there is an equilibrium in which the seller sets a deterministic p. Recall that if $\mu\in(a+p-1/(4\kappa), a+p+1/(4\kappa))$, the consumer's learning has support $\{a+p-1/(4\kappa), a+p+1/(4\kappa)\}$. Otherwise, the consumer does not learn and either buys with certainty (if μ is sufficiently high) or takes her outside option with certainty. That leaves us with three cases.

Suppose first that $p\geq\mu-a+1/(4\kappa)$, so that the consumer does not learn and takes her outside option (on path). The seller's profit is 0. Evidently, it can deviate by charging some price $\tilde{p}\in(0,\mu-a)$, which yields it profit $\tilde{p}>0$.

Second, suppose that $p \in (\mu - a - 1/(4\kappa), \mu - a + 1/(4\kappa))$. Conditional on the consumer purchasing, the seller's profit is p. Suppose it deviates to a price $\tilde{p} := p + 1/(8\kappa)$. Evidently,

$$v_H - \tilde{p} = a + p + \frac{1}{4\kappa} - \left(p + \frac{1}{8\kappa}\right) = a + \frac{1}{8\kappa} > a \ ,$$

so the consumer still purchases with the same probability at a strictly higher price, yielding a strictly higher profit for the seller.

Third, suppose that $p \leq \mu - a - 1/(4\kappa)$, so that the consumer does not learn and purchases from the seller. Evidently, the firm's profit is p. However, it can deviate to some $\tilde{p} = \mu - a - \epsilon$, which is strictly higher than p and 0 provided $\epsilon < \min\{1/(4\kappa), \mu - a\}$.

We have established the necessity of $\mu \leq a$ for the existence of a pure-strategy equilibrium. Now let us deduce the no-trade result.

Let $\mu \leq a$, and suppose for the sake of contradiction that there exists a pure-strategy equilibrium in which trade occurs. Consequently, either $p \in (\mu - a - 1/(4\kappa), \mu - a + 1/(4\kappa))$ or $p \leq \mu - a - 1/(4\kappa)$. The latter is clearly impossible, since the seller is making negative profits. In the first case, we must have $p \leq \mu - a + 1/(4\kappa)$. Clearly, if $\mu - a + 1/(4\kappa) \leq 0$, there are no pure-strategy equilibria with trade. Thus, let us assume $\mu - a + 1/(4\kappa) > 0$. If there is trade (with positive probability), then $0 \leq p < \mu - a + 1/(4\kappa)$. But then the firm can raise its price slightly, thereby increasing its profit.

Finally, let us construct pure strategy equilibria in which there is no trade. Evidently, we must have $p \geq \mu - a + 1/(4\kappa)$. The consumer will not learn and so (on-path) the consumer will take her outside option. On path, the firm gets 0 and any deviation would yield it at most 0 since $\mu \leq a$. ■

A.7 Theorem 5.2 Proof

Proof. Again, for convenience, we assume that there is a single representative consumer. From Lemma 5.1, we know that there exist no pure strategy equilibria in this region. As noted in the main part of the text, given the consumer's equilibrium distribution over values, F, the monopolist's profit is

$$\hat{\Pi}(p) = p\left(1 - F(p + a)\right) \ ,$$

which must equal some constant $\lambda \geq 0$ for all p in the support of its mixed strategy since it is mixing. Denote the support of its mixed strategy by $\left[\underline{p}, \bar{p}\right]$, where $\underline{p} \geq 0$. Given the monopolist's mixed strategy over prices G, the consumer's distribution (F) must solve

$$\max_F \int_{-\infty}^{\infty} \left\{ \int_{\underline{p}}^{x-a} (x-p)\, dG(p) + \int_{x-a}^{\bar{p}} a\, dG(p) - \kappa (x-\mu)^2 \right\} dF(x) \ .$$

It is also helpful to write down the consumer's payoff as a function of the realized value:

$$V(x) = \begin{cases} a - \kappa (x-\mu)^2, & x < a + \underline{p} \\ a\left(1 - G(x-a)\right) + \int_{\underline{p}}^{x-a} (x-p)\, dG(p) - \kappa (x-\mu)^2, & a + \underline{p} \leq x \leq a + \bar{p} \\ x - \mathbb{E}_G[p] - \kappa (x-\mu)^2, & a + \bar{p} \leq x \end{cases} .$$

Concavifying V yields the consumer's optimal distribution(s). Our first pair of claims ensures that the upper and lower bounds of the support of any equilibrium distributions over values and prices differ from each other (respectively) only by the value of the outside option.

Claim A.1. *In any equilibrium, the upper bound of the support of the consumer's distribution, $\bar{x}$, satisfies $\bar{x} = a + \bar{p}$.*

Proof. It is easy to see that we cannot have $\bar{x} < a + \bar{p}$. Indeed, the monopolist's profit from any price $p > \bar{x} - a$ is 0, whereas it can always obtain a strictly positive profit by setting price $p' \in (0, \mu - a)$. Can we have $\bar{x} > \bar{p} + a$? In this case the consumer's optimal distribution above $\bar{p} + a$ must consist of a single mass point, since information is costly. However, then we have $F(\bar{p} + a) = F(\bar{p} + a + \eta)$ for all (sufficiently small) strictly positive η, contradicting the optimality of the monopolist's distribution over prices. ■

Claim A.2. *In any equilibrium, the lower bound of the support of the consumer's distribution, $\underline{x}$, satisfies $\underline{x} = a + \underline{p}$.*

Proof. That $\underline{x} \leq a + \underline{p}$ is also straightforward. Otherwise, the monopolist obtains a strictly higher profit from $\underline{p} + \epsilon$ than from $\underline{p}$, a violation. Can we have $\underline{x} < \underline{p} + a$? In this case the consumer's optimal distribution below $\bar{p} + a$ must consist of a single mass point, since information is costly. Moreover, F must have no support on $(\underline{x}, \tilde{x})$, where $\tilde{x} > \underline{p} + a$ since V

is strictly concave on $\left(-\infty, \underline{p}+a\right)$. However, this means that the monopolist strictly prefers price $\underline{p}+\eta$ to $\underline{p}$ since demand is locally perfectly inelastic in a neighborhood of $\underline{p}$. ■

We now establish that the consumer's distribution over values has full support.

Claim A.3. *In any equilibrium the distributions chosen by the consumer and the monopolist have full support on* $[\underline{x}, \bar{x}]$ *and* $\left[\underline{p}, \bar{p}\right]$*, respectively.*

Proof. Suppose that there is a gap in the support of the consumer's distribution, i.e., $F(x_1) = F(x_2)$ for some $x_2 > x_1$. Then, $\hat{\Pi}(p) < \hat{\Pi}(p_2)$ for all $p \in (p_1, p_2)$, where $p_i = x_i - a$. Accordingly, the firm does not charge any price p in this interval, i.e., $G(p_1) = G^{-}(p_2)$.[16] Then,

$$V(x_1) = a(1 - G(p_1)) + \int_{\underline{p}}^{p_1} (x_1 - p)\, dG(p) - \kappa (x_1 - \mu)^2 \;,$$

and

$$\begin{aligned} V(x_2) &= a(1 - G(p_2)) + \int_{\underline{p}}^{p_2} (x_2 - p)\, dG(p) - \kappa (x_2 - \mu)^2 \\ &= a(1 - G(p_1)) + \int_{\underline{p}}^{p_1} (x_2 - p)\, dG(p) - \kappa (x_2 - \mu)^2 \end{aligned}.$$

By the strict convexity of the cost function, F cannot be optimal for the consumer, who could strictly benefit by having value $x' := \lambda x_1 + (1-\lambda) x_2$ for any $\lambda \in (0,1)$ in the support of F, rather than x_1 and x_2.

The analysis of the previous paragraph also implies that the monopolist's distribution can have no gaps in its support. ■

The next claim establishes that the monopolist's distribution over prices may not have mass points and neither does the consumer's distribution over values except possibly at the upper bound of its support.

Claim A.4. *In any equilibrium the distribution chosen by the monopolist has no mass points and the distribution chosen by the consumer has no mass points except possibly at* $\bar{x}$*.*

[16]We define $G^{-}(p) := \sup_{w<p} G(w)$.

Proof. Suppose for the sake of contradiction that F has a mass point at some value $\hat{x} < \bar{x}$ and let $\hat{p} = \hat{x} + a$. Evidently,

$$\hat{\Pi}(\hat{p} - \epsilon) = (\hat{p} - \epsilon)(1 - F(\hat{x} - \epsilon)) > (\hat{p} + \epsilon)(1 - F(\hat{x} + \epsilon)) = \hat{\Pi}(\hat{p} + \epsilon) \ ,$$

for sufficiently small $\epsilon > 0$. This implies that the support of G has a gap, contradicting our previous claim.

Because the consumer's distribution over values has full support, the consumer's payoff as a function of value x, $V(x)$, must be affine. The distribution over prices cannot have a mass point, since otherwise V would have a discrete change in its slope induced by the mass point. ■

At last we may derive the unique equilibrium. Setting $\hat{\Pi}(p) = \lambda$, we obtain

$$F(x) = 1 - \frac{\lambda}{x - a} \ .$$

Since F has no mass points on $[\underline{x}, \bar{x})$, $\lambda = \underline{x} - a$. Moreover, since $\mathbb{E}_F[x] = \mu$, we have

$$1 + \ln\left\{\frac{\bar{x} - a}{\underline{x} - a}\right\} = \frac{\mu - a}{\underline{x} - a} \ . \tag{A2}$$

Because V is affine,

$$a(1 - G(x - a)) + \int_{\underline{x}-a}^{x-a} (x - p)\, dG(p) - \kappa(x - \mu)^2 = \alpha x + \beta \ , \forall\, x \in [\underline{x}, \bar{x}] \ , \tag{A3}$$

where $\alpha, \beta \in \mathbb{R}$. We may differentiate both sides and do some algebra, which yields

$$G(p) = 2\kappa(a + p - \mu) + \alpha \ ,$$

and substituting this back in to Equation A3, we obtain

$$\alpha = 2\kappa(\mu - \underline{x}) \ , \tag{A4}$$

and

$$\beta = \kappa\left(\underline{x}^2 - \mu^2\right) + a \ . \tag{A5}$$

Since $G(\bar{p}) = 1$,

$$\alpha + 2\kappa(\bar{p} + a - \mu) = 1 \ . \tag{A6}$$

Thus, combining Equations $A4$ and $A6$, we get

$$\bar{p} = \underline{p} + \frac{1}{2\kappa} \ . \tag{A7}$$

Substituting in for $\bar{p}$ in Equation $A2$ via Equation $A7$ and using the fact that $\underline{x} = a + \underline{p}$, we have

$$\ln\left\{1 + \frac{1}{2\kappa \underline{p}}\right\} = \frac{\mu - a}{\underline{p}} - 1 \ .$$

The firm's profit is clearly $\underline{p}$ since if it charges that price it is purchased from with certainty, and the consumer's payoff is just $V(\mu)$. ■

A.8 Lemma 5.3 Proof

Proof. Let us define the function $\iota\left(\kappa, \underline{p}, \mu\right)$ as

$$\iota\left(\underline{p}_M, \kappa, \mu\right) := \underline{p}_M \ln\left\{1 + \frac{1}{2\kappa \underline{p}_M}\right\} + \underline{p}_M - \mu + a \ .$$

Evidently, ι is twice continuously differentiable in each of its arguments on $\mathbb{R}^3_{++}$.

Claim A.5. *ι has a strictly positive root if and only if $\mu > a$. This root, $\underline{p}_M$, is the unique positive root and lies in the interval $\underline{p}_M \in (0, \mu - a)$.*

Proof. Directly,

$$\iota'\left(\underline{p}_M\right) = -\frac{1}{2\kappa \underline{p}_M + 1} + \ln\left\{\frac{1}{2\kappa \underline{p}_M} + 1\right\} + 1, \quad \text{and} \quad \iota''\left(\underline{p}_M\right) = -\frac{1}{\underline{p}_M\left(2\kappa \underline{p}_M + 1\right)^2} \ .$$

Evidently, $\iota''\left(\underline{p}_M\right) < 0$ for all $\underline{p}_M > 0$. We claim that ι is strictly increasing in $\underline{p}_M$ for $\underline{p}_M > 0$. For all such $\underline{p}_M$, $\iota'\left(\underline{p}_M\right)$ is strictly decreasing in $\underline{p}_M$. This, plus the fact that $\lim_{\underline{p}_M \to \infty} \iota'\left(\underline{p}_M\right) = 1$, implies that ι is strictly increasing in $\underline{p}_M$ for all $\underline{p}_M > 0$.

Then, we see that as $\underline{p}_M \searrow 0$, $\iota \searrow a - \mu$, which is strictly negative if and only if $\mu > a$; and as $\underline{p}_M \nearrow \mu - a$, $\iota \nearrow (\mu - a)\ln\{1 + 1/(2\kappa(\mu - a))\}$, which is strictly positive if and only if $\mu > a$. Thus, if $\mu > a$, $\iota\left(\underline{p}_M\right)$ has a unique positive root at some $\underline{p}_M \in (0, \mu - a)$. ■

Directly, $\iota'(\mu) = -1$ and

$$\iota'(\kappa) = -\frac{\underline{p}_M}{2\underline{p}_M \kappa^2 + \kappa} < 0 \ ;$$

and so by the implicit function theorem, $\underline{p}_M'(\mu) > 0$ and $\underline{p}_M'(\kappa) > 0$. ■

A.9 Lemma 5.4 Proof

Proof. Recall that the monopolist's profit is just the lower bound for the price, $\underline{p}_M$, and so its profit is strictly increasing in κ and μ.

The consumer's payoff is $\hat{\Phi} = \kappa\left(\mu - a - \underline{p}_M\right)^2 + a$. Accordingly,

$$\hat{\Phi}'(\mu) = 2\kappa\left(\mu - a - \underline{p}_M\right)\left(1 - \underline{p}_M'(\mu)\right) .$$

Then,

$$\underline{p}_M'(\mu) = \frac{1}{-\frac{1}{2\kappa\underline{p}_M+1} + \ln\left\{\frac{1}{2\kappa\underline{p}_M} + 1\right\} + 1} = \frac{z+1}{(z+1)\ln\left\{\frac{z+1}{z}\right\} + z} ,$$

where $z := 2\underline{p}_M\kappa$. Moreover, by the definition of $\underline{p}_M$

$$2\kappa\left(\mu - a - \underline{p}_M\right) = z\ln\left\{\frac{z+1}{z}\right\} .$$

Consequently, $\hat{\Phi}'(\mu)$ reduces to

$$\hat{\Phi}'(\mu) = z\ln\left\{\frac{z+1}{z}\right\}\left(1 - \frac{z+1}{(z+1)\ln\left\{\frac{z+1}{z}\right\} + z}\right) ,$$

which is strictly positive for all $z > 0$.

Similarly,

$$\hat{\Phi}'(\kappa) = \left(\mu - a - \underline{p}_M\right)^2 - 2\kappa\left(\mu - a - \underline{p}_M\right)\underline{p}_M'(\kappa) ,$$

which is positive if and only if

$$\kappa\left(\mu - a - \underline{p}_M\right) - 2\kappa^2\underline{p}_M'(\kappa) \geq 0 , \qquad (A8)$$

since $\mu - a > \underline{p}_M$. Then,

$$\underline{p}_M'(\kappa) = \frac{\frac{\underline{p}_M}{2\underline{p}_M\kappa^2+\kappa}}{-\frac{1}{2\kappa\underline{p}_M+1} + \ln\left\{\frac{1}{2\kappa\underline{p}_M} + 1\right\} + 1} = \left(\frac{\underline{p}_M}{\kappa}\right)\frac{1}{(z+1)\ln\left\{\frac{z+1}{z}\right\} + z} ,$$

where again $z := 2\underline{p}_M\kappa$. Inequality A8 is, therefore, equivalent to

$$\frac{1}{2}\ln\left\{\frac{z+1}{z}\right\} - \frac{1}{(z+1)\ln\left\{\frac{z+1}{z}\right\} + z} \geq 0 , \qquad (A9)$$

which is equivalent to requiring that $\kappa\underline{p}_M \lesssim .337$. ■

A.10 Proposition 5.6 Proof

Proof. First, we show that $\underline{p}_M$ is strictly concave in κ and $\underline{p}$ is strictly convex in κ.

Claim A.6. *$\underline{p}_M$ is strictly concave in κ.*

Proof. By the implicit function theorem (and dropping the subscript M),

$$\underline{p}''(\kappa) = \frac{-\iota_{\underline{p}}^2 \iota_{\kappa\kappa} + 2\iota_{\underline{p}}\iota_{\kappa}\iota_{\kappa\underline{p}} - \iota_{\kappa}^2 \iota_{\underline{p}\underline{p}}}{\iota_{\underline{p}}^3} \text{ .}$$

Since $\iota_{\underline{p}} > 0$ this is strictly negative if and only if

$$-\iota_{\underline{p}}^2 \iota_{\kappa\kappa} + 2\iota_{\underline{p}}\iota_{\kappa}\iota_{\kappa\underline{p}} - \iota_{\kappa}^2 \iota_{\underline{p}\underline{p}} < 0 \text{ .} \tag{A10}$$

Let us go through each term one by one. First,

$$-\iota_{\underline{p}}^2 \iota_{\kappa\kappa} = -\left(\frac{z}{z+1} + \ln\left\{\frac{1}{z} + 1\right\}\right)^2 \frac{\underline{p}\,(2z+1)}{\kappa^2\,(z+1)^2} \text{ ,}$$

where $z := 2\kappa\underline{p}$. Second,

$$2\iota_{\underline{p}}\iota_{\kappa}\iota_{\kappa\underline{p}} = 2\left(\frac{z}{z+1} + \ln\left\{\frac{1}{z} + 1\right\}\right)\frac{\underline{p}}{\kappa^2}\left(\frac{1}{(z+1)^3}\right) \text{ .}$$

Third,

$$-\iota_{\kappa}^2 \iota_{\underline{p}\underline{p}} = \frac{\underline{p}}{\kappa^2}\left(\frac{1}{(z+1)^4}\right)$$

Summing these together (and dividing out by positive terms), Inequality A13 holds if and only if

$$2\ln\left(\frac{1}{z}+1\right)(1-2z) + \frac{2z+1}{z+1}(1-z) - \left(\ln\left(\frac{1}{z}+1\right)\right)^2 (2z+1) < 0 \text{ ,} \tag{A11}$$

which holds for all strictly positive z. ■

Claim A.7. *$\underline{p}$ is strictly convex in κ.*

Proof. Let us define the function $\varphi\left(\kappa, \underline{p}, c\right)$ as

$$\varphi\left(\kappa, \underline{p}, c\right) := \underline{p}\ln\left\{1 + \frac{1}{2\kappa\underline{p}}\right\} - \sqrt{\frac{c}{\kappa}} \text{ .}$$

Evidently, φ is twice continuously differentiable in each of its variables on $\mathbb{R}^3_{++}$.

We shall break protocol slightly in this proof and use results that are derived (not much) later on in the paper (that φ is strictly increasing in $\underline{p}$ for $\underline{p} > 0$, which we establish in Section A.11). Mirroring the proof of the previous claim, we need to show that

$$-\varphi_{\underline{p}}^2\varphi_{\kappa\kappa} + 2\varphi_{\underline{p}}\varphi_{\kappa}\varphi_{\kappa\underline{p}} - \varphi_{\kappa}^2\varphi_{\underline{p}\underline{p}} > 0 \,. \tag{A12}$$

Again, let us go through each term one by one. First,

$$\begin{aligned}
-\varphi_{\underline{p}}^2\varphi_{\kappa\kappa} &= -\left(\ln\left\{1+\frac{1}{x}\right\} - \frac{1}{1+x}\right)^2\left(\frac{x(2x+1)}{2(1+x)^2\kappa^3} - \frac{3\sqrt{c\kappa}}{4\kappa^3}\right) \\
&= -\left(\ln\left\{1+\frac{1}{x}\right\} - \frac{1}{1+x}\right)^2\left(\frac{x(2x+1)}{2(1+x)^2\kappa^3} - \frac{3x\ln\left\{1+\frac{1}{x}\right\}}{8\kappa^3}\right),
\end{aligned}$$

where $x := 2\kappa\underline{p}$ and since we must have

$$\sqrt{c\kappa} = \frac{x}{2}\ln\left\{1+\frac{1}{x}\right\}.$$

Second,

$$\begin{aligned}
2\varphi_{\underline{p}}\varphi_{\kappa}\varphi_{\kappa\underline{p}} &= -2\left(\ln\left\{1+\frac{1}{x}\right\} - \frac{1}{1+x}\right)\left(\frac{\sqrt{c\kappa}}{2\kappa^2} - \frac{x}{2(x+1)\kappa^2}\right)\left(\frac{1}{\kappa(x+1)^2}\right) \\
&= -2\left(\ln\left\{1+\frac{1}{x}\right\} - \frac{1}{1+x}\right)\left(\frac{x\ln\left\{1+\frac{1}{x}\right\}}{4\kappa^2} - \frac{x}{2(x+1)\kappa^2}\right)\left(\frac{1}{\kappa(x+1)^2}\right).
\end{aligned}$$

Third,

$$-\varphi_{\kappa}^2\varphi_{\underline{p}\underline{p}} = \left(\frac{x\ln\left\{1+\frac{1}{x}\right\}}{4\kappa^2} - \frac{x}{2(x+1)\kappa^2}\right)^2\left(\frac{1}{\underline{p}}\right)\frac{1}{(1+x)^2}.$$

Summing these together (and dividing out by positive terms), Inequality A12 holds if and only if

$$3\ln^3\left(\frac{1}{x}+1\right)(x+1)^3 + \ln^2\left(\frac{1}{x}+1\right)(x+1)(-14x-13) + 19\ln\left(\frac{1}{x}+1\right)(x+1) - 8 > 0\,,$$

which holds for all strictly positive x. ■

Next, we establish that $\underline{p}_M$ ranges from 0 to some finite positive number and that $\underline{p}$ is U-shaped in κ and explodes at $\kappa = 0$ and $\kappa = 1/(4c)$.

Claim A.8. $\infty > \lim_{\kappa\nearrow 1/(4c)}\underline{p}_M > 0$ *and* $\lim_{\kappa\searrow 0}\underline{p}_M = 0$.

Proof. The first statement in the claim follows immediately from the analysis in Section A.8. Suppose for the sake of contradiction that $\underline{p}_M > L$ for some constant $L > 0$ for all $\kappa > 0$. Thus,

$$\lim_{\kappa \searrow 0} \underline{p}_M \ln\left\{1 + \frac{1}{2\kappa \underline{p}_M}\right\} + \underline{p}_M - \mu \geq \lim_{\kappa \searrow 0} L \ln\left\{1 + \frac{1}{2\kappa L}\right\} + L - \mu = \infty \; .$$

Since ι is continuous and strictly decreasing in κ, this means that there is some $\hat{\kappa} > 0$ such that for all $0 < \kappa < \hat{\kappa}$, $\iota(\kappa) > 0$, a contradiction. ∎

Claim A.9. $\lim_{\kappa \nearrow 1/(4c)} \underline{p} = \lim_{\kappa \searrow 0} \underline{p} = \infty$.

Proof. As is shown in Section A.11, $\underline{p}$ is strictly increasing in κ for all $\kappa \in (\tilde{\kappa}, 1/(4c))$, where $\tilde{\kappa} \in (0, 1/(4c))$. Suppose for the sake of contradiction that $\underline{p}$ is bounded above by $K < \infty$ for all $\kappa \in (\tilde{\kappa}, 1/(4c))$. We have

$$\lim_{\kappa \nearrow \frac{1}{4c}} \underline{p} \ln\left\{1 + \frac{1}{2\kappa \underline{p}}\right\} - \sqrt{\frac{c}{\kappa}} \leq \lim_{\kappa \nearrow \frac{1}{4c}} K \ln\left\{1 + \frac{1}{2\kappa K}\right\} - \sqrt{\frac{c}{\kappa}} = K \ln\left\{1 + \frac{2c}{K}\right\} - 2c < 0 \; ,$$

a contradiction.

Similarly, $\underline{p}$ is strictly decreasing in κ for all $\kappa \in (0, \tilde{\kappa})$. Suppose for the sake of contradiction that $\underline{p}$ is bounded above by $M < \infty$ for all $\kappa \in (0, \tilde{\kappa})$. We have

$$\lim_{\kappa \searrow 0} \frac{\underline{p} \ln\left\{1 + \frac{1}{2\kappa \underline{p}}\right\}}{\sqrt{\frac{c}{\kappa}}} \leq \lim_{\kappa \searrow 0} \frac{M \ln\left\{1 + \frac{1}{2\kappa M}\right\}}{\sqrt{\frac{c}{\kappa}}} = 0 < 1 \; ,$$

a contradiction. ∎

Next, (setting $a = 0$)

$$\iota\left(\mu - \sqrt{\frac{c}{\kappa}}\right) = \left(\mu - \sqrt{\frac{c}{\kappa}}\right) \ln\left\{1 + \frac{1}{2\kappa\left(\mu - \sqrt{\frac{c}{\kappa}}\right)}\right\} - \sqrt{\frac{c}{\kappa}} =: \rho(\mu) \; ,$$

which is equal to 0 for $\kappa > 0$ if and only if

$$x(\mu) \ln\left(1 + \frac{1}{x(\mu)}\right) = 2\sqrt{c\kappa} \; ,$$

where $x(\mu) := 2\kappa\mu - 2\sqrt{c\kappa}$. Then, it is easy to see that the left-hand side of this equation is strictly increasing and strictly concave in μ and converges to 1 as μ goes to ∞. On the other hand, the right-hand side is a constant (for c and κ fixed) and is strictly less than 1, since $\kappa < 1/(4c)$. Thus, for all (allowed) κ and c, there exists a μ such that the equilibrium involves active search. Furthermore, if $\mu \leq (<) \sqrt{c/\kappa}$, $\underline{p}_M$ is obviously (strictly) greater than $\mu - \sqrt{c/\kappa}$.

Finally, since $\underline{p}_M$ is continuous and strictly increasing in μ for $\mu > 0$ and ranges from 0 to some positive value, $\underline{p}$ is U-shaped and explodes at its endpoints, and $\underline{p}_M$ is strictly concave in κ and $\underline{p}$ is strictly convex in κ, the result follows. ■

A.11 Proposition 5.7 Proof

Proof. Let us begin with the following claim.

Claim A.10. *$\varphi\left(\underline{p}\right)$ has a real root at some $\underline{p} > 0$ if and only if $c < 1/(4\kappa)$. This root is unique.*

Proof. First, we argue that φ cannot have a root at some $\underline{p} < 0$. Directly,

$$\varphi'\left(\underline{p}\right) = \ln\left\{1 + \frac{1}{2\kappa\underline{p}}\right\} - \frac{1}{1 + 2\kappa\underline{p}}, \quad \text{and} \quad \varphi''\left(\underline{p}\right) = -\frac{1}{\underline{p}\left(2\kappa\underline{p} + 1\right)^2} .$$

Evidently, $\varphi''\left(\underline{p}\right) > 0$ for all $\underline{p} < 0$. Consequently, since $\varphi'\left(\underline{p}\right)$ is strictly increasing in $\underline{p}$ for all $\underline{p} \in (-\infty, 0)$ and $\lim_{\underline{p} \to -\infty} \varphi'\left(\underline{p}\right) = 0$, φ is strictly increasing in $\underline{p}$ for all $\underline{p} \in (-\infty, 0)$. Since $\lim_{\underline{p} \nearrow 0} \iota\left(\underline{p}_M\right) = -\sqrt{c/\kappa} < 0$, we conclude that there is no root.

Second, $\lim_{\underline{p} \searrow 0} \varphi\left(\underline{p}\right) = -\sqrt{c/\kappa}$, thereby eliminating 0 as a candidate root.

Third, we establish that φ is strictly increasing in $\underline{p}$ for strictly positive $\underline{p}$. Since $\varphi'\left(\underline{p}\right)$ is strictly decreasing in $\underline{p}$ and $\lim_{\underline{p} \to \infty} \varphi'\left(\underline{p}\right) = 0$, φ is strictly increasing in $\underline{p}$ for all $\underline{p}$.

Fourth, as $\underline{p} \searrow 0$, $\varphi \searrow -\sqrt{c/\kappa} < 0$; and as $\underline{p} \nearrow +\infty$, $\varphi \nearrow 1/(2\kappa) - \sqrt{c/\kappa}$. Thus, $\varphi\left(\underline{p}\right)$ has a positive real root if and only if $c < 1/(4\kappa)$. It is unique. ■

Evidently, $\varphi'(c) < 0$. Then, by the implicit function theorem,

$$\underline{p}'(c) = -\frac{\varphi'(c)}{\varphi'\left(\underline{p}\right)} > 0,$$

where we used the result from Claim A.10 that $\varphi'\left(\underline{p}\right) > 0$.

Next,

$$\varphi'(\kappa) = \frac{\sqrt{c\kappa}}{2\kappa^2} - \frac{1}{2\left(\frac{1}{2\underline{p}\kappa} + 1\right)\kappa^2} ,$$

which is positive if and only if

$$\underline{p} \leq \frac{\sqrt{c}}{2\sqrt{\kappa} - 2\kappa\sqrt{c}} .$$

Then,

$$\begin{aligned}\varphi\left(\frac{\sqrt{c}}{2\sqrt{\kappa} - 2\kappa\sqrt{c}}\right) \geq 0 \quad &\Leftrightarrow \quad \frac{\sqrt{c}}{2\sqrt{\kappa} - 2\kappa\sqrt{c}} \ln\left\{1 + \frac{1}{2\kappa\frac{\sqrt{c}}{2\sqrt{\kappa}-2\kappa\sqrt{c}}}\right\} - \sqrt{\frac{c}{\kappa}} \geq 0 \\ &\Leftrightarrow \quad -\ln\left\{\sqrt{c\kappa}\right\} - 2 + 2\sqrt{c\kappa} \geq 0\end{aligned} .$$

By the implicit function theorem, $\underline{p}$ is decreasing in κ if and only if $\varphi'(\kappa) \geq 0$, if and only if $h(c,\kappa) := \ln\left\{\sqrt{c\kappa}\right\} + 2 - 2\sqrt{c\kappa} \leq 0$. Note that h has a unique root at $\tilde{\kappa} \in (0, 1/(4c))$. ■

A.12 Lemma 5.9 Proof

Proof. Let us begin with the result pertaining to the explicit search cost c. Directly,

$$\hat{\Phi}'(c) = 1 - \underline{p}'(c) - \frac{1}{2\sqrt{c\kappa}} < 0 ,$$

since $\underline{p}'(c) > 0$ and $c < 1/(4\kappa)$. Next,

$$\hat{\Phi}'(\kappa) = -\underline{p}'(\kappa) + \frac{1}{2\kappa}\sqrt{\frac{c}{\kappa}} \geq 0 \quad \Leftrightarrow \quad \frac{\varphi'(\kappa)}{\varphi'\left(\underline{p}\right)} \geq -\frac{1}{2\kappa}\sqrt{\frac{c}{\kappa}} .$$

Substituting in for $\varphi'(\kappa)$ and $\varphi'\left(\underline{p}\right)$, this holds if and only if

$$\frac{\frac{\sqrt{c\kappa}}{2\kappa^2} - \frac{1}{2\left(\frac{1}{2\underline{p}\kappa}+1\right)\kappa^2}}{\ln\left\{1 + \frac{1}{2\kappa\underline{p}}\right\} - \frac{1}{1+2\kappa\underline{p}}} \geq -\frac{1}{2\kappa}\sqrt{\frac{c}{\kappa}} \quad \Leftrightarrow \quad 2\underline{p}\kappa\left(\sqrt{c\kappa} - 1\right) + \sqrt{c\kappa}\ln\left(1 + \frac{1}{2\kappa\underline{p}}\right)\left(1 + 2\underline{p}\kappa\right) \geq 0 .$$

Enacting a change of variable with $x := \kappa\underline{p}$ and substituting in using $\varphi = 0$, the latter inequality becomes

$$w(x) := \left(1 + \frac{1}{2x}\right)\ln\left\{1 + \frac{1}{2x}\right\} + 1 - \frac{1}{x\ln\left\{1 + \frac{1}{2x}\right\}} \geq 0 .$$

Note that x is strictly increasing in κ for all $\kappa \in (\tilde{\kappa}, 1/(4c))$. Define $\tilde{x} := \tilde{\kappa}\underline{p}(\tilde{\kappa})$ and observe that by construction, $w(\tilde{x}) > 0$. Moreover, w is strictly decreasing in x and $\lim_{x\to\infty} w(x) = -1$. Appealing to the intermediate value theorem, we deduce the result. ■

A.13 Proposition 6.1 Proof

Proof. Recall

$$\iota\left(\underline{p},\kappa,\mu\right) := \underline{p}\ln\left\{1+\frac{1}{2\kappa\underline{p}}\right\}+\underline{p}-\mu\ ,$$

(where we have set $a=0$), which is strictly increasing in $\underline{p}$.

First, let $\mu \geq 3/(4\kappa)$. We have

$$\iota(p) < \iota\left(\mu-\frac{1}{2\kappa}\right) = \left(\mu-\frac{1}{2\kappa}\right)\ln\left\{1+\frac{1}{2\kappa\left(\mu-\frac{1}{2\kappa}\right)}\right\}+\left(\mu-\frac{1}{2\kappa}\right)-\mu\ ,$$

for all $p < \mu - 1/(2\kappa)$. The right-hand side of this expression is weakly positive if and only if

$$(z-1)\ln\left\{\frac{z}{z-1}\right\}-1 \geq 0\ ,$$

where $z := 2\kappa\mu$, which never holds. Thus, $\underline{p} \geq \mu - 1/(2\kappa)$. We also have

$$\iota(p) > \iota\left(\mu-\frac{1}{4\kappa}\right) = \left(\mu-\frac{1}{4\kappa}\right)\ln\left\{1+\frac{1}{2\kappa\left(\mu-\frac{1}{4\kappa}\right)}\right\}+\left(\mu-\frac{1}{4\kappa}\right)-\mu\ ,$$

for all $p > \mu - 1/(4\kappa)$. The right-hand side of this expression is weakly positive if and only if

$$(z-1)\ln\left\{\frac{z+1}{z-1}\right\}-1 \geq 0\ , \tag{A13}$$

where $z := 4\kappa\mu$. Since $\mu \geq 3/(4\kappa)$, $z \geq 3$ and so Inequality $A13$ holds. Consequently, $\underline{p} \leq \mu - 1/(4\kappa)$.

Second, let $3/(4\kappa) \geq \mu > 0$. We have

$$\iota(p) < \iota\left(\frac{\mu}{2}-\frac{1}{8\kappa}\right) = \left(\frac{\mu}{2}-\frac{1}{8\kappa}\right)\ln\left\{1+\frac{1}{2\kappa\left(\frac{\mu}{2}-\frac{1}{8\kappa}\right)}\right\}+\left(\frac{\mu}{2}-\frac{1}{8\kappa}\right)-\mu\ ,$$

for all $p < \mu/2 - 1/(8\kappa)$. The right-hand side of this expression is weakly positive if and only if

$$(z-1)\ln\left\{\frac{z+3}{z-1}\right\}-1-z \geq 0\ ,$$

where $z := 4\kappa\mu$, which never holds. Thus, $\underline{p} \geq \mu/2 - 1/(8\kappa)$. Finally, we also have

$$\iota(p) > \iota\left(\frac{(1+4\kappa\mu)^2}{32\kappa}\right) = \frac{(1+4\kappa\mu)^2}{32\kappa}\ln\left\{1+\frac{1}{2\kappa\frac{(1+4\kappa\mu)^2}{32\kappa}}\right\}+\frac{(1+4\kappa\mu)^2}{32\kappa}-\mu\ ,$$

for all $p > {}^{(1+4\kappa\mu)^2}/_{(32\kappa)}$. The right-hand side of this expression is weakly positive if and only if

$$(1+z)^2 \ln\left\{\frac{(1+z)^2+16}{(1+z)^2}\right\} + (1+z)^2 - 8z \geq 0 \ ,$$

where $z := 4\kappa\mu$. This clearly holds. Consequently, $\underline{p} < {}^{(1+4\kappa\mu)^2}/_{(32\kappa)}$. ∎

A.14 Proposition 6.2 Proof

Proof. First, we establish an auxiliary result.

Claim A.11. *If $\mu \geq \underline{p} + \sqrt{c/\kappa}$, then $\underline{p} > \sqrt{c/\kappa} - 1/(4\kappa)$.*

Proof. If $c \leq 1/(16\kappa)$, the result is immediate, so let $c > 1/(16\kappa)$. Since φ is strictly increasing in $\underline{p}$ for all $\underline{p} > 0$, we have

$$\varphi\left(\underline{p}\right) \leq \varphi\left(\sqrt{\frac{c}{\kappa}} - \frac{1}{4\kappa}\right) = \left(\sqrt{\frac{c}{\kappa}} - \frac{1}{4\kappa}\right) \ln\left\{\frac{4\sqrt{c\kappa}+1}{4\sqrt{c\kappa}-1}\right\} - \sqrt{\frac{c}{\kappa}} \ ,$$

for all $\underline{p} \leq \mu - \sqrt{c/\kappa}$. The right hand side of this expression is weakly positive if and only if

$$f(x) := (x-1)\ln\left\{\frac{x+1}{x-1}\right\} - x \geq 0 \ ,$$

where $x := 4\sqrt{c\kappa}$. Note that x takes values in the interval $(1,2)$, and it is easy to see that $f < 0$ for all such x. Therefore, $\underline{p} > \sqrt{c/\kappa} - 1/(4\kappa)$. ∎

Second, we establish the result for high μ.

Claim A.12. *If $\mu \geq \underline{p} + \sqrt{c/\kappa}$, then $\Phi_l > \hat{\Phi}$.*

Proof.

$$\Phi_l > \hat{\Phi} \quad \Leftrightarrow \quad \mu + \frac{1}{4\kappa} + c - 2\sqrt{\frac{c}{\kappa}} > \mu - \underline{p} + c - \sqrt{\frac{c}{\kappa}} \quad \Leftrightarrow \quad \underline{p} > \sqrt{\frac{c}{\kappa}} - \frac{1}{4\kappa} \ ,$$

which follows from Claim A.11. ∎

Third, we establish the result for moderate μ.

Claim A.13. *If $\underline{p} + \sqrt{c/\kappa} > \mu \geq 2\sqrt{c/\kappa} - 1/(4\kappa)$, then $\Phi_l > \hat{\Phi}_M$.*

Proof. Using the conditions of the claim,

$$\Phi = \mu + \frac{1}{4\kappa} + c - 2\sqrt{\frac{c}{\kappa}} \geq c > \kappa\left(\mu - \underline{p}\right)^2 = \hat{\Phi}_M \ .$$

■

Finally, if $\underline{p} + \sqrt{c/\kappa} > \mu$ and $2\sqrt{c/\kappa} - 1/(4\kappa) > \mu$, the result is derived in the proof of Proposition 6.1. ■

A.15 Proposition 6.3 Proof

Proof. First, we establish the result for high μ.

Claim A.14. *If $\mu \geq \underline{p} + \sqrt{c/\kappa}$, then $\underline{p} > 2c$.*

Proof. By Claim A.11,if $\mu \geq \underline{p} + \sqrt{c/\kappa}$, then $\underline{p} > \sqrt{c/\kappa} - 1/(4\kappa)$, i.e., if there is active search when prices are hidden, there must be active search when prices are observable. Since φ is strictly increasing in $\underline{p}$ for all $\underline{p} > 0$, we have

$$\varphi\left(\underline{p}\right) \leq \varphi(2c) = 2c \ln\left\{\frac{4\kappa c + 1}{4\kappa c}\right\} - \sqrt{\frac{c}{\kappa}} \ ,$$

for all $\underline{p} \leq 2c$. The right hand side of this expression is negative if and only if

$$x \ln\left\{\frac{x+1}{x}\right\} - \sqrt{x} < 0 \ ,$$

where $x := 4\kappa c$. This clearly holds for all $x \in (0,1)$. ■

Thus, if the mean is sufficiently high so that there is active search when prices are hidden, firms prefer that prices be hidden. Second, we establish that for medium μ, firms prefer hidden before learning if and only if μ is sufficiently high.

Lemma A.15. *Let $\mu > 0$. There exist κ, μ and c that generate $\underline{p} + \sqrt{c/\kappa} > \mu \geq 2\sqrt{c/\kappa} - 1/(4\kappa)$, such that $\underline{p} > 2c$, $\underline{p} = 2c$, or $\underline{p} < 2c$.*

Proof. Note, that the condition for this result is that there is active search if and only if prices are observable. Since ι is strictly decreasing in μ it suffices to show that there exist values of the parameters such that $\underline{p} = 2c$, since at the two endpoints ($\underline{p} + \sqrt{c/\kappa}$ and $2\sqrt{c/\kappa} - 1/(4\kappa)$) firms strictly prefer active or hidden search, respectively. Directly, we have

$$\iota(2c) = 2c \ln\left\{\frac{4\kappa c + 1}{4\kappa c}\right\} + 2c - \mu \ .$$

This must equal 0 at equilibrium, i.e.,

$$\mu = 2c \ln\left\{\frac{4\kappa c + 1}{4\kappa c}\right\} + 2c \ .$$

Consequently, we must have

$$2c + \sqrt{\frac{c}{\kappa}} > 2c \ln\left\{\frac{4\kappa c + 1}{4\kappa c}\right\} + 2c \geq 2\sqrt{\frac{c}{\kappa}} - \frac{1}{4\kappa} \ ,$$

which holds if and only if

$$\sqrt{x} > x \ln\left\{\frac{x+1}{x}\right\} \geq 2\sqrt{x} - x - \frac{1}{2} \ ,$$

where $x := 4\kappa c$. This holds for all $x \in (0,1)$. ■

Third, if $\underline{p} + \sqrt{c/\kappa} > \mu$ and $2\sqrt{c/\kappa} - 1/(4\kappa) > \mu$, then the proof of Proposition 6.1 implies that $\underline{p} < (1 + 4\kappa\mu)^2/(32\kappa)$. That is, if there is no active search for either timing regime, firms prefer that prices be observable.

Finally, if $0 \geq \mu > -1/(4\kappa)$, the firms obviously prefer that prices are not hidden since otherwise there is no trade. ■