Search and Competition with Endogenous Investigations
Supplementary Appendix

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1 Mathematica Code for Result Verification

This supplement provides Mathematica code for verifying the results for some of the auxiliary (algebra-heavy) results in Jain and Whitmeyer (2021).

1.1 Proposition 4.4 Calculations

As stated in the paper, there is active search and information acquisition provided

$$\mu \geq 2\sqrt{\frac{c}{\kappa}} - \frac{1}{4\kappa} \,. \tag{A1}$$

Case 2 refers to $\mu > 0$ and $\mu/4 < c < \mu/3$, Case 3 refers to $\mu \geq 0$ and $c \geq \mu/3$, and Case 4 refers to $\mu < 0$.

In Case 2, when does Inequality A1 hold?

```
Reduce[\[Mu] > 0 && \[Mu]/4 < c < \[Mu]/3 &&
  0 < \[Kappa] < 1/(4 c) && \[Mu] >=
   2 Sqrt[c/\[Kappa]] - 1/(4 \[Kappa])]
```

In Case 3, when does Inequality A1 hold?

```
Reduce[\[Mu] > 0 && c >= \[Mu]/3 &&
  0 < \[Kappa] < 1/(4 c) && \[Mu] >=
   2 Sqrt[c/\[Kappa]] - 1/(4 \[Kappa])]
```

In Case 4, when does Inequality A1 hold?

```
Reduce[\[Mu] < 0  && \[Mu] >=
   2 Sqrt[c/\[Kappa]] - 1/(4 \[Kappa]), {\[Mu], c, \[Kappa]}]
```

1.2 Lemma 5.3 Calculations

As defined in the paper,

$$\iota\left(\underline{p}_M, \kappa, \mu\right) := \underline{p}_M \ln\left\{1 + \frac{1}{2\kappa \underline{p}_M}\right\} + \underline{p}_M - \mu + a \,.$$

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Computing $\iota'\left(\underline{p}_M\right)$.

```
D[Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
    Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] + Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M], Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M] ]
```

Verifying that ι' is strictly decreasing in $\underline{p}_M$.

```
Reduce[D[D[Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
        Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] + Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M], Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M] ], Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M] ] > 0 && Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M] > 0]
```

Computing $\iota'(\kappa)$.

```
D[Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
   Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])], \[Kappa]  ]
```

1.3 Lemma 5.4 Calculations

As stated in the paper

$$\frac{1}{2}\ln\left\{\frac{z+1}{z}\right\} - \frac{1}{(z+1)\ln\left\{\frac{z+1}{z}\right\}+z} \geq 0\ . \qquad (A9)$$

Verifying the last step.

```
Reduce[1/2 Log[1 + 1/z] - 1/((z + 1) Log[1 + 1/z] + z) >= 0 &&
  z > 0, Reals]
N[Root[{-2 + Log[1 + #^(-1)] # + Log[1 + #^(-1)]^2 (1 + #) &,
   0.67311392763327999759032728476285112329`20.298128281937895}]]
```

1.4 Claim A.6 Calculations

As stated in the paper,

$$\underline{p}''(\kappa) = \frac{-\iota_{\underline{p}}^2\iota_{\kappa\kappa} + 2\iota_{\underline{p}}\iota_{\kappa}\iota_{\kappa\underline{p}} - \iota_{\kappa}^2\iota_{\underline{p}\underline{p}}}{\iota_{\underline{p}}^3}\ .$$

Computing the first term (of three) in the numerator of this expression.

```
FullSimplify[-D[D[Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
        Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] + Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M], \[Kappa] ], \[Kappa] ] D[
    Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
       Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] + Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M], Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M] ]^2]
```

Computing the second term (of three) in the numerator of the expression for $\underline{p}''(\kappa)$.

```
FullSimplify[2 D[Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
      Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] + Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M], Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M] ] D[Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
      Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] + Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M], \[Kappa]  ] D[D[Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
       Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] + Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M], \[Kappa] ], Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]]]
```

Computing the third term (of three) in the numerator of the expression for $\underline{p}''(\kappa)$.

```
FullSimplify[-D[Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
        Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] + Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M], \[Kappa]  ]^2 D[D[Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
       Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] + Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M], Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M] ], Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]]]
```

As defined in the paper,

$$2\ln\left(\frac{1}{z}+1\right)(1-2z)+\frac{2z+1}{z+1}(1-z)-\left(\ln\left(\frac{1}{z}+1\right)\right)^2(2z+1)<0\ . \tag{A11}$$

Verifying Inequality $A11$.

```
Reduce[2 Log[
      1/z + 1] (1 - 2 z) + (2 z + 1)/(z + 1) (1 -
       z) - (Log[1/z + 1])^2 (2 z + 1) > 0 && z > 0, Reals]
```

1.5 Claim A.7 Calculations

As defined in the paper,

$$\varphi\left(\kappa,\underline{p},c\right):=\underline{p}\ln\left\{1+\frac{1}{2\kappa\underline{p}}\right\}-\sqrt{\frac{c}{\kappa}}\ .$$

Computing $\varphi_{\underline{p}}^2\varphi_{\kappa\kappa}$.

```
FullSimplify[-D[Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
        Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] - Sqrt[c/\[Kappa]],
     Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]]^2 D[D[Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M]
```

```
       Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] -
     Sqrt[c/\[Kappa]], \[Kappa]], \[Kappa]]]

Reduce[((-1 + Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] +
      2 \[Kappa] Log[1 + 1/(2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])^2 (3 c -
      4 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\),
       M] (-3 c + Sqrt[
         c/\[Kappa]] + (-3 c + 4 Sqrt[c/\[Kappa]]) \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M])))/(
   4 Sqrt[c/\[Kappa]] \[Kappa]^3 (1 + 2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\),
       M])^4) != -(Log[1 + 1/x] -
        1/(1 + x))^2 (x (2 x + 1)/(2 (1 + x)^2 \[Kappa]^3) -
      3 x Log[1 + 1/x]/(8 \[Kappa] ^3)) && x == 2 \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(p\), \(_\)]\), M] &&
  Sqrt[c \[Kappa]] == x/2 Log[1 + 1/x] && \[Kappa] Subscript[
\!\(\*UnderscriptBox[\(\(\ \)\(p\)\), \(_\)]\), M] > 0  &&
  c > 0, Reals]
```

Computing $2\varphi_{\underline{p}}\varphi_{\kappa}\varphi_{\kappa\underline{p}}$.

```
FullSimplify[2 D[
\!\(\*UnderscriptBox[\(p\), \(_\)]\) Log[1 + 1/(2 \[Kappa]
\!\(\*UnderscriptBox[\(p\), \(_\)]\))] - Sqrt[c/\[Kappa]],
\!\(\*UnderscriptBox[\(p\), \(_\)]\)] D[
\!\(\*UnderscriptBox[\(p\), \(_\)]\) Log[1 + 1/(2 \[Kappa]
\!\(\*UnderscriptBox[\(p\), \(_\)]\))] -
    Sqrt[c/\[Kappa]], \[Kappa]] D[D[
\!\(\*UnderscriptBox[\(p\), \(_\)]\) Log[1 + 1/(2 \[Kappa]
\!\(\*UnderscriptBox[\(p\), \(_\)]\))] - Sqrt[c/\[Kappa]], \[Kappa]],
\!\(\*UnderscriptBox[\(p\), \(_\)]\)]]

FullSimplify[-2 (Log[1 + 1/x] -
    1/(1 + x)) (x Log[1 + 1/x]/(4 \[Kappa]^2) -
    x/(2 (x + 1) \[Kappa]^2)) (1/(\[Kappa] (x + 1)^2))]

Reduce[Exists[{\[Kappa],
\!\(\*UnderscriptBox[\(p\), \(_\)]\), c, x}, \[Kappa]
\!\(\*UnderscriptBox[\(\(\ \)\(p\)\), \(_\)]\) > 0  &&
   c > 0 && -(((Sqrt[c/\[Kappa]] + 2 (-1 + Sqrt[c/\[Kappa]] \[Kappa])
\!\(\*UnderscriptBox[\(p\), \(_\)]\)) (-1 + Log[1 + 1/(2 \[Kappa]
\!\(\*UnderscriptBox[\(p\), \(_\)]\))] +
        2 \[Kappa] Log[1 + 1/(2 \[Kappa]
\!\(\*UnderscriptBox[\(p\), \(_\)]\))]
\!\(\*UnderscriptBox[\(p\), \(_\)]\)))/(\[Kappa]^2 (1 + 2 \[Kappa]
\!\(\*UnderscriptBox[\(p\), \(_\)]\))^4)) != -((
     x (-2 + (1 + x) Log[1 + 1/x]) (-1 + (1 + x) Log[1 + 1/x]))/(
     2 (1 + x)^4 \[Kappa]^3)) && x == 2 \[Kappa]
\!\(\*UnderscriptBox[\(p\), \(_\)]\) &&
   Sqrt[c \[Kappa]] == x/2 Log[1 + 1/x]], Reals]
```

Computing $\varphi_{\kappa}^{2}\varphi_{\underline{p}\underline{p}}$.

```
FullSimplify[-(D[
\!\(\*UnderscriptBox[\(p\), \(_\)]\) Log[1 + 1/(2 \[Kappa]
```

```
\!\(\*UnderscriptBox[\(p\), \(_\)]\))] -
       Sqrt[c/\[Kappa]], \[Kappa]])^2*D[D[
\!\(\*UnderscriptBox[\(p\), \(_\)]\) Log[1 + 1/(2 \[Kappa]
\!\(\*UnderscriptBox[\(p\), \(_\)]\))] - Sqrt[c/\[Kappa]],
\!\(\*UnderscriptBox[\(p\), \(_\)]\)],
\!\(\*UnderscriptBox[\(p\), \(_\)]\)]]

FullSimplify[(x Log[1 + 1/x]/(4 \[Kappa]^2) -
     x/(2 (x + 1) \[Kappa]^2))^2 (1/
\!\(\*UnderscriptBox[\(p\), \(_\)]\)) (1/(1 + x)^2)]

Reduce[(c + 2 (c - Sqrt[c/\[Kappa]]) \[Kappa]
\!\(\*UnderscriptBox[\(p\), \(_\)]\))^2/(4 c \[Kappa]^3
\!\(\*UnderscriptBox[\(p\), \(_\)]\) (1 + 2 \[Kappa]
\!\(\*UnderscriptBox[\(p\), \(_\)]\))^4) != (
   x^2 (-2 + (1 + x) Log[1 + 1/x])^2)/(16 (1 + x)^4 \[Kappa]^4
\!\(\*UnderscriptBox[\(p\), \(_\)]\)) && x == 2 \[Kappa]
\!\(\*UnderscriptBox[\(p\), \(_\)]\) &&
  Sqrt[c \[Kappa]] == x/2 Log[1 + 1/x] && \[Kappa]
\!\(\*UnderscriptBox[\(\(\ \)\(p\)\), \(_\)]\) > 0  && c > 0, Reals]
```

Verifying the last inequality.

```
Reduce[3 Log[1/x + 1]^3 (x + 1)^3 +
    Log[1/x + 1]^2 (x + 1) (-14 x - 13) + 19 Log[1/x + 1] (x + 1) -
    8 <= 0 && x > 0, Reals]
```

2 Proposition 6.1 Calculations

As stated in the paper,

$$(z-1)\ln\left\{\frac{z+1}{z-1}\right\}-1 \geq 0\,. \tag{A13}$$

Verifying that Inequality $A13$ holds.

```
Reduce[(z - 1) Log[(z + 1)/(z - 1)] - 1 < 0 && z >= 3]
```

Establishing that we cannot have $\underline{p} < \mu/2 - 1/(8\kappa)$.

```
Reduce[(z - 1) Log[(z + 3)/(z - 1)] - 1 - z >= 0, Reals]
```

3 Claim A.11 Calculations

As defined in the paper,

$$f(x) := (x-1)\ln\left\{\frac{x+1}{x-1}\right\}.$$

Verifying that $f(x) < 0$ for all $x \in (1,2)$.

```
Reduce[(x - 1) Log[(x + 1)/(x - 1)] - x >= 0 && 1 < x < 2]
```

4 Lemma A.15 Calculations

Verifying

$$\sqrt{x} > x\ln\left\{\frac{x+1}{x}\right\} \geq 2\sqrt{x} - x - \frac{1}{2}\,.$$

```
Reduce[Sqrt[x] > x Log[1 + 1/x] >= 2 Sqrt[x] - x - 1/2 &&
  0 < x < 1, Reals]
```

References

Vasudha Jain and Mark Whitmeyer. Search and competition with endogenous investigations. *Mimeo*, 2021.